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KNOXVILLE, TENN. 37916

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ESTIMATION OF THE OPERATING CHARACTERISTICS WHEN THE TEST INFORMATION OF THE OLD TEST IS NOT CONSTANT I: RATIONALE

ABSTRACT

Many combinations of a method and an approach for estimating the operating characteristics of the graded item responses, without assuming any mathematical forms, have been produced. In these methods, we need a set of items whose characteristics are known, or Old Test, which has a large, constant amount of test information throughout the interval of latent trait of our interest. In the present paper, the rationale is presented to generalize these methods so that they are made applicable when the test information of the Old Test is not constant. Both the transformation-free character of the maximum likelihood estimator and the method of moments for fitting a polynomial as the least squares solution play important roles in this rationale.

The research was conducted at the principal investigator's laboratory, 409 Austin Peay Hall, Department of Psychology, University of Tennessee, Knoxville, Tennessee. The computer programming was greatly assisted by Philip S. Livingston. Other people who helped the author for this research working in her laboratory include Paul S. Changas, Dete Furlan, C. I. Bonnie Chen, Robert L. Trestman, and Pamela Welch.

TABLE OF CONTENTS

I Introduction

There have been produced many combinations of a method and an approach for estimating the operating characteristics of graded item responses (Samejima, 1972), which have two distinguishing characteristics such that:

(1) No prior mathematical forms are assumed for the resulting operating characteristics,

and:

(2) A relatively small number of subjects, say, several hundred, are needed for the basic data for the estimation.

(cf. Samejima, 1977c, 1977d, 1978a, 1978b, 1978c, 1978d, 1978e, 1978f.) We can categorize these methods and approaches as follows.

[A] Approaches:

- (i) Histogram Ratio Approach
- (ii) Curve Fitting Approach
- (iii) Conditional P.D.F. Approach
  - (a) Simple Sum Procedure
  - (b) Weighted Sum Procedure
  - (c) Proportioned Sum Procedure
- (iv) Bivariate P.D.F. Approach

[B] Methods:

- (i) Two-Parameter Beta Method
- (ii) Pearson System Method

(iii) Normal Approach Method

It has been found out that all of these combinations of an approach and a method provide us with good estimations of operating characteristics, although each combination has its own merits as well as its relative shortcomings when compared with the other combinations.

These combinations of a method and an approach have also such additional characteristics that:

(3) We need a set of items whose operating characteristics are known, in order to estimate the operating characteristics of "unknown" items;

and

(4) Such a set of "known" items, which is called Old Test, must provide us with a substantially large and constant amount of test information for the interval of latent trait of our interest.

A typical situation which possesses these characteristics in itself is the tailored testing situation, where we have an item pool from which an optimal subset of test items is selected and presented to a specific examinee. When we wish to add new items to the item pool, all we need is to use a fixed amount of test information as the criterion for terminating the presentation of new items to every individual subject (cf. 1977a, 1977b). Thus Old Test in this situation is not a single set of test items, but a combination of as many subtests as the number of examinees who provided us with the basic

data for the estimation of the operating characteristics. We notice that, though these features, (3) and (4), are suitable in the tailored testing situation, they will restrict the applicability of the estimation methods in the paper-and-pencil testing situation, where we are forced to use a fixed set of test items.

In some situations, efforts have been put upon the elimination of feature (3) using equivalent items and Constant Information Model, a new family of models, and so forth, so that we shall be able to use the methods without depending upon the Old Test (cf. Samejima, 1979a, 1979b, 1979c). We note, however, that, even if we may have to depend upon the Old Test in estimating the operating characteristics of "new items," the applicability of the methods will be enhanced enormously under any circumstances, if we can eliminate the requirement of the constant test information, which is stated in (4), i.e., <u>if we can use a set of "known" items whose test information function is not constant</u> for the interval of ability of our interest, as Old Test. Fortunately, this expansion of the methods is relatively easy and straight-forward, at least, in theory.

In the present paper, the rationale behind this generalization of the methods will be presented and discussed. In so doing, the transformation-free character of the maximum likelihood estimator (Samejima, 1969) takes an essential role. The method of moments for fitting a polynomial, which proved to be also the least squares solution (Samejima and Livingston, 1979), plays another important role.

The procedures presented in this paper will be applied in the simulation study in the near future, and will be published as separate papers, in order to investigate how the theory works in practice.

II Transformation of Latent Trait

Let θ be the latent trait, or ability, which assumes any real number, such that

$$(2.1) \qquad -\infty < \theta < \infty \ .$$

Let g $(=1,2,\ldots,n)$ be an item, and x_g $(=0,1,\ldots,m_g)$ be a graded item response (Samejima, 1969, 1972), which is reduced to the binary item response when $m_g=1$. The operating characteristic of the graded item response is denoted by $P_{x_g}(\theta)$, which is the conditional probability with which the examinee obtains the item score, or provides us with the graded item response, x_g , given ability θ . Two typical examples of this operating characteristic are those in the normal ogive model and in the logistic model, defined on the graded response level (Samejima, 1972). The item response information function, $I_{x_g}(\theta)$, is defined as the negative of the second partial derivative of the natural logarithm of the operating characteristic, such that

$$(2.2) \qquad I_{x_g}(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_{x_g}(\theta) \ ,$$

and the item information function is the regression of the item response information function on ability θ , which can be written as

$$(2.3) \qquad I_g(\theta) = \sum_{x_g=0}^{m_g} I_{x_g}(\theta)\, P_{x_g}(\theta) \ .$$

This item information function can be considered as an index of local

accuracy of estimation of θ provided by the item g, if the item response information function assumes a positive value for every item response x_g (Samejima, 1973b), as is the case of the normal ogive and the logistic models on the graded response level (cf. Samejima, 1969, 1972, 1973a).

Let V be the response pattern of the graded item responses, such that

$$(2.4) \qquad V = (x_1, x_2, \ldots, x_n)' \ .$$

The operating characteristic of the response pattern V, which is the conditional probability with which the examinee obtains the response pattern V, given θ, and is denoted by $P_V(\theta)$, can be written, in virtue of the assumption of local independence (Lord and Novick, 1968), by the formula

$$(2.5) \qquad P_V(\theta) = \prod_{x_g \epsilon V} P_{x_g}(\theta) \ ,$$

and the response pattern information function, $I_V(\theta)$, is the negative of the second partial derivative of the natural logarithm of the operating characteristic of the response pattern, such that

$$(2.6) \qquad I_V(\theta) = -\frac{\partial^2}{\partial\theta^2} \log P_V(\theta)$$

$$= \sum_{x_g \epsilon V} I_{x_g}(\theta) \ .$$

The test information function, $I(\theta)$, is defined as the regression of the response pattern information function on ability θ , such that

$$(2.7) \qquad I(\theta) = \sum_{V} I_V(\theta)\, P_V(\theta) \quad .$$

It has been shown both on the dichotomous and the graded response levels that this test information function can be written as the sum total of the item information functions, such that

$$(2.8) \qquad I(\theta) = \sum_{g=1}^{n} I_g(\theta)$$

(Birnbaum, 1968; Samejima, 1969). We can prove from (2.3) that the item information function is non-negative in nature, regardless of the values of the item response information functions. By virtue of (2.8), therefore, the test information function, $I(\theta)$, is also non-negative in nature, and is used as an index of local accuracy of estimation of ability θ provided by the test. Note, however, that this index is meaningless unless the item response information function assumes a non-negative value for every item response x_g , since, otherwise, the existence of the unique maximum likelihood estimate is not assured for every possible response pattern, as is the case in the three-parameter normal ogive and logistic models (cf. Samejima, 1969, 1972, 1973b).

Let τ be a function of θ , such that

$$\tau = \tau(\theta) \quad , \tag{2.9}$$

which is strictly increasing in θ . The operating characteristic, $P^*_{x_g}(\tau)$, of the item response x_g defined for the transformed latent trait τ equals the original operating characteristic, $P_{x_g}(\theta)$, which is obvious from its definition as the conditional probability. Thus we can write

$$P^*_{x_g}(\tau) = P^*_{x_g}[\tau(\theta)] = P_{x_g}(\theta) \ . \tag{2.10}$$

From (2.2) and (2.10), we can write for the item response information function, $I^*_{x_g}(\tau)$, such that

$$\begin{aligned} I^*_{x_g}(\tau) &= -\frac{\partial^2}{\partial\tau^2}\log P^*_{x_g}(\tau) \\ &= I_{x_g}(\theta)\,[\frac{d\theta}{d\tau}]^2 - \frac{\partial}{\partial\theta}\log P_{x_g}(\theta)\,.\,\frac{d^2\theta}{d\tau^2} \ . \end{aligned} \tag{2.11}$$

From this result, we have for the item information function $I^*_g(\tau)$,

$$\begin{aligned} I^*_g(\tau) &= \sum_{x_g=0}^{m_g} I^*_{x_g}(\tau)\,P^*_{x_g}(\tau) \\ &= I_g(\theta)\,[\frac{d\theta}{d\tau}]^2, \end{aligned} \tag{2.12}$$

since

$$\sum_{x_g=0}^{m_g} \frac{\partial}{\partial\theta} P_{x_g}(\theta) = 0 \ . \tag{2.13}$$

It can be seen that, with the response pattern V, we obtain similar results, such that

$$(2.14) \qquad P^*_V(\tau) = P^*_V[\tau(\theta)] = P_V(\theta)$$

for the operating characteristic, $P^*_V(\tau)$, and

$$(2.15) \qquad I^*_V(\tau) = I_V(\theta)\, [\frac{d\theta}{d\tau}]^2 - \frac{\partial}{\partial\theta} \log P_V(\theta) . \frac{d^2\theta}{d\tau^2}$$

for the information function, $I^*_V(\tau)$. We can write for the test information function $I^*(\tau)$ either from (2.15) or from (2.12) such that

$$(2.16) \qquad I^*(\tau) = I(\theta)\, [\frac{d\theta}{d\tau}]^2$$

and, since τ is a strictly increasing function of θ, we have

$$(2.17) \qquad [I^*(\tau)]^{1/2} = [I(\theta)]^{1/2} \frac{d\theta}{d\tau} .$$

The maximum likelihood estimate, $\hat{\theta}$, of ability θ, which is based upon the response pattern V, can be obtained by using the operating characteristics $P_V(\theta)$ as the likelihood function. In a similar manner, the corresponding maximum likelihood estimate, $\hat{\tau}$, can be obtained by using $P^*_V(\tau)$ as the likelihood function. By virtue of the transformation-free character of the maximum likelihood estimator, however, this second maximum likelihood estimate can also be obtained by the direct transformation of $\hat{\theta}$, such that

(2.18) $\hat{\tau} = \tau(\hat{\theta})$

(cf. Samejima, 1969).

Note that (2.18) has a great deal of practical importance, especially when the transformation, $\tau(\ )$, is given by a relatively simple formula. Since in most cases there exists no sufficient statistic for the response pattern V, the maximum likelihood estimate, $\hat{\tau}$, must be obtained through a numerical process, using the basic function $A^*_{x_g}(\tau)$, which is defined by

(2.19) $$A^*_{x_g}(\tau) = \frac{\partial}{\partial\tau} \log P^*_{x_g}(\tau)$$

(cf. Samejima, 1969, 1972). Substituting (2.10) into (2.19), we can write

(2.20) $$\begin{aligned} A^*_{x_g}(\tau) &= \frac{d\theta}{d\tau} \frac{\partial}{\partial\theta} \log P_{x_g}(\theta) \\ &= \frac{d\theta}{d\tau} A_{x_g}(\theta) \quad , \end{aligned}$$

where $A_{x_g}(\theta)$ is the basic function of the item response x_g defined with respect to θ. Since the derivative, $\frac{d\theta}{d\tau}$, is usually of a complicated form, it is not easy to program the process so that we shall be able to obtain the maximum likelihood estimate $\hat{\tau}$ as the solution to the equation,

(2.21) $$\sum_{x_g \in V} A^*_{x_g}(\tau) = 0 \quad .$$

It is much easier, therefore, to obtain the maximum likelihood $\hat{\theta}$

from the basic function, $\Lambda_{x_g}(\theta)$, and then obtain $\hat{\tau}$ through the formula (2.18).

III <u>Latent Trait Providing a Constant Test Information for a Specific Test</u>

Here we assume that the test information function, $I(\theta)$, of a specific test of our interest is <u>not</u> constant for the interval $[\underline{\theta}, \bar{\theta}]$. We attempt to transform the latent trait θ to τ , <u>in such a way that the resultant test information function,</u> $I^*(\tau)$, <u>be constant for the interval,</u> $[\underline{\tau}, \bar{\tau}]$, where

$$(3.1) \qquad \begin{cases} \underline{\tau} = \tau(\underline{\theta}) \\ \bar{\tau} = \tau(\bar{\theta}) \end{cases} .$$

Let C^2 denote this desired, constant amount of test information. From (2.17) we can write

$$(3.2) \qquad \frac{d\tau}{d\theta} = C^{-1} [I(\theta)]^{1/2} .$$

Now we obtain from (3.2) for the transformation of θ to τ

$$(3.3) \qquad \tau = C^{-1} \int [I(\theta)]^{1/2} d\theta + d ,$$

where d is an arbitrary constant.

Thus it has been shown that, as far as the square root of test information function is integrable, we can always transform the latent trait θ to another scale, τ , by means of (3.3), in such a way that the resultant test information, $I^*(\tau)$, be constant. A problem arises, however, when $[I(\theta)]^{1/2}$ is not integrable, or its integral

provides us with a highly complicated form, as is usually the case. Perhaps the best practical solution for this problem is the use of the method of moments.

It has been shown by Samejima and Livingston (Samejima and Livingston, 1979) that the polynomial provided by the method of moments to approximate any given function is also its least squares solution, which is an appropriate characteristic for the present purpose. It has also been demonstrated that, in fitting such a polynomial, it is important to find an optimal interval of the independent variable for the computation of the moments in order to obtain a well-fitted function. If we succeed in obtaining such a polynomial, we can write

$$(3.4) \qquad [I(\theta)]^{1/2} \doteq \sum_{k=0}^{m} \alpha_k \theta^k \quad ,$$

where k is the degree of the polynomial. Substituting (3.4) into (3.3), we obtain

$$(3.5) \qquad \tau \doteq C^{-1} \sum_{k=0}^{m} \alpha_k \, (k+1)^{-1} \theta^{k+1} + d$$

$$= \sum_{k=0}^{m+1} \alpha_k^* \theta^k \quad ,$$

where

$$(3.6) \qquad \alpha_k^* \begin{cases} = d & k = 0 \\ = (Ck)^{-1} \alpha_{k-1} & k = 1,2,\ldots,m+1 \quad . \end{cases}$$

The transformation of θ to τ can be made, therefore, through a polynomial of degree $(m+1)$, which is quite simple.

For the purpose of illustration, we hypothesize two tests, whose test information functions are not constant. Each of these two tests consists of twenty-five graded test items with $m_g = 2$. Since they are both subsets of the thirty-five test items of Old Test used in the previous studies, we shall call them Subtests 1 and 2, respectively. All these test items follow the normal ogive model, whose operating characteristics are given by

$$(3.7) \qquad P_{x_g}(\theta) = [2\pi]^{-1/2} \int_{a_g(\theta-b_{x_g+1})}^{a_g(\theta-b_{x_g})} \exp[-u^2/2]\, du$$

where a_g (>0) is the item discrimination parameter and b_{x_g} is the item response difficulty parameter, which satisfies

$$(3.8) \qquad -\infty = b_0 < b_1 \ \ldots \ < b_{m_g} < b_{m_g+1} = \infty \ .$$

These item parameters are shown in Tables 3-1 and 3-2.

The item information function, $I_g(\theta)$, for each item of Subtests 1 and 2 was obtained through (3.7), (2.2) and (2.3), and the two test information functions, $I(\theta)$, were obtained through (2.8). Figures 3-1 and 3-2 present the square roots of the test information functions thus obtained by solid curves, for Subtests 1 and 2, respectively.

Taking $\underline{\theta} = -3.0$ and $\bar{\theta} = 3.0$, the moments about the

TABLE 3-1

Item Discrimination Parameters of the Twenty-Five Items of Each of Subtests 1 and 2

Item g	a_g	Subtest 1	Subtest 2
1	1.8		x
2	1.9		x
3	2.0		x
4	1.5		x
5	1.6		x
6	1.4	x	x
7	1.9	x	x
8	1.8	x	x
9	1.6	x	x
10	2.0	x	x
11	1.5	x	x
12	1.7	x	x
13	1.5	x	
14	1.4	x	
15	2.0	x	
16	1.6	x	
17	1.8	x	
18	1.7	x	
19	1.9	x	
20	1.7	x	
21	1.5	x	
22	1.8	x	
23	1.4	x	x
24	1.9	x	x
25	2.0	x	x
26	1.6	x	x
27	1.7	x	x
28	1.4	x	x
29	1.9	x	x
30	1.6	x	x
31	1.5		x
32	1.7		x
33	1.8		x
34	2.0		x
35	1.4		x

TABLE 3-2

Two Item Difficulty Parameters of Each Item of Subtests 1 and 2

Item g	b_1	b_2	Subtest 1	Subtest 2
1	-4.75	-3.75		x
2	-4.50	-3.50		x
3	-4.25	-3.25		x
4	-4.00	-3.00		x
5	-3.75	-2.75		x
6	-3.50	-2.50	x	x
7	-3.00	-2.00	x	x
8	-3.00	-2.00	x	x
9	-2.75	-1.75	x	x
10	-2.50	-1.50	x	x
11	-2.25	-1.25	x	x
12	-2.00	-1.00	x	x
13	-1.75	-0.75	x	
14	-1.50	-0.50	x	
15	-1.25	-0.25	x	
16	-1.00	0.00	x	
17	-0.75	0.25	x	
18	-0.50	0.50	x	
19	-0.25	0.75	x	
20	0.00	1.00	x	
21	0.25	1.25	x	
22	0.50	1.50	x	
23	0.75	1.75	x	x
24	1.00	2.00	x	x
25	1.25	2.25	x	x
26	1.50	2.50	x	x
27	1.75	2.75	x	x
28	2.00	3.00	x	x
29	2.25	3.25	x	x
30	2.50	3.50	x	x
31	2.75	3.75		x
32	3.00	4.00		x
33	3.25	4.25		x
34	3.50	4.50		x
35	3.75	4.75		x

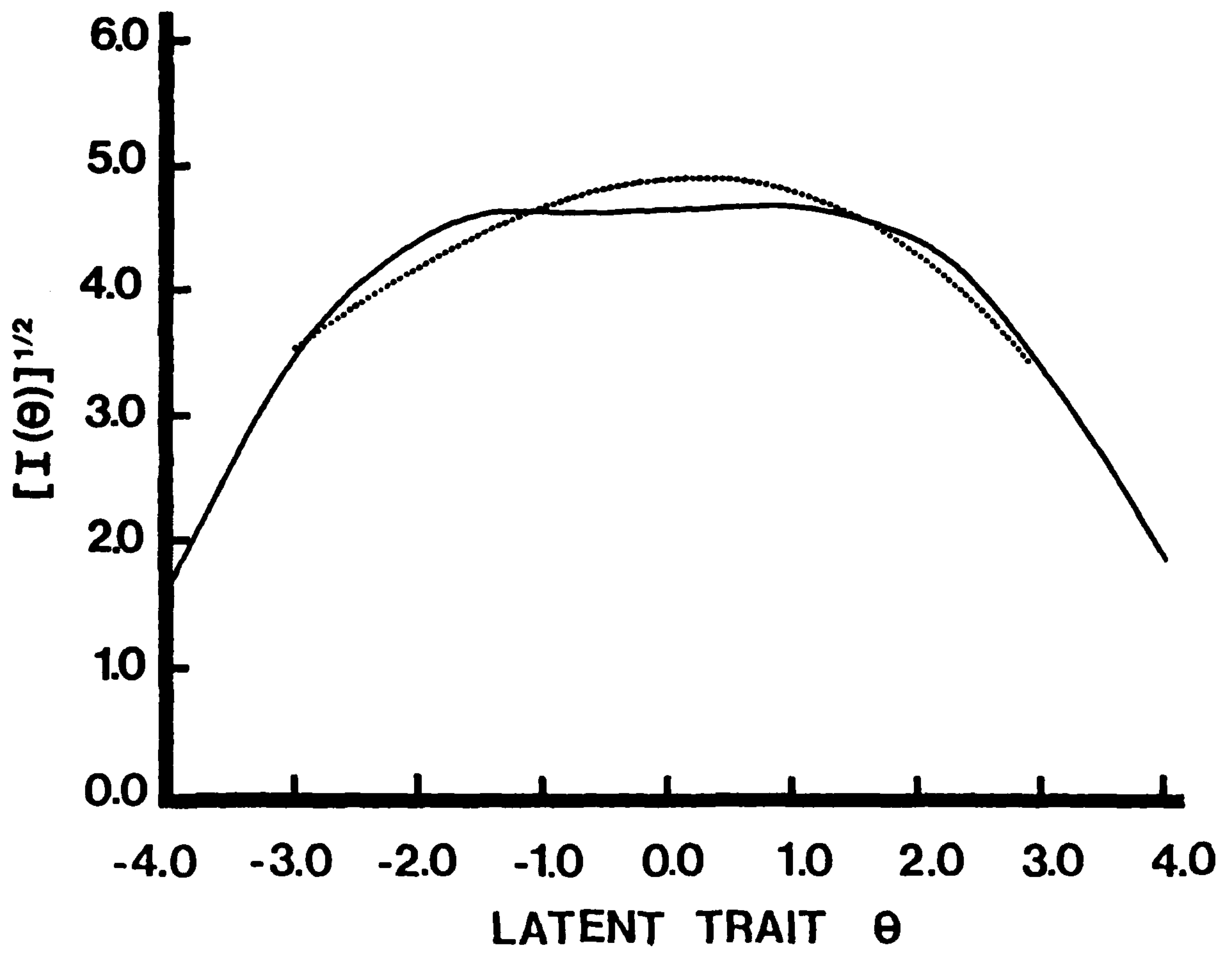

FIGURE 3-1

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) and the Polynomial of Degree 3 (Dotted Line), Which Was Fitted by the Method of Moments with [-3.0, 3.0] As the Interval of θ .

Subtest 1

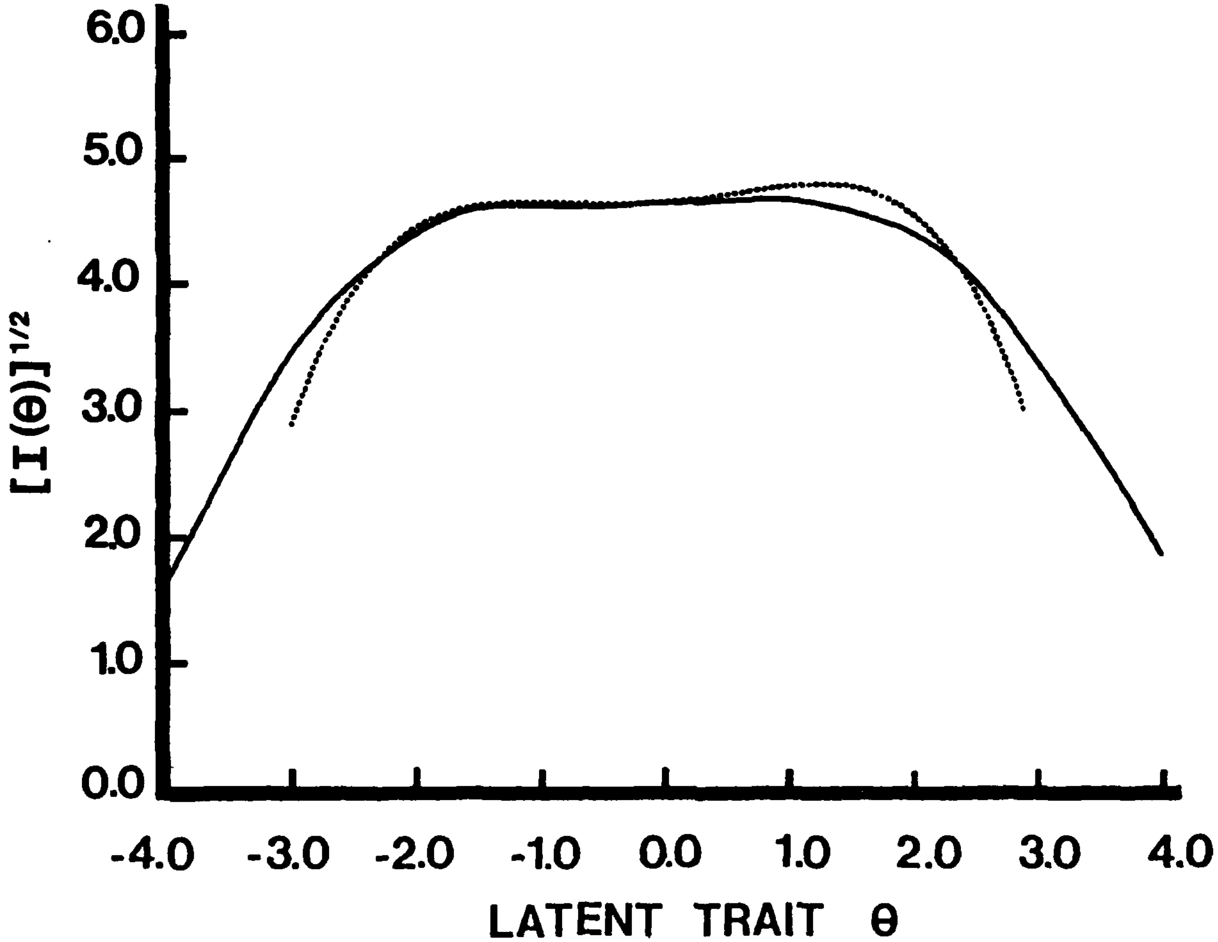

FIGURE 3-1 (Continued): Subtest 1, Polynomial of Degree 4, $[\underline{\theta}, \bar{\theta}] = [-3.0, 3.0]$.

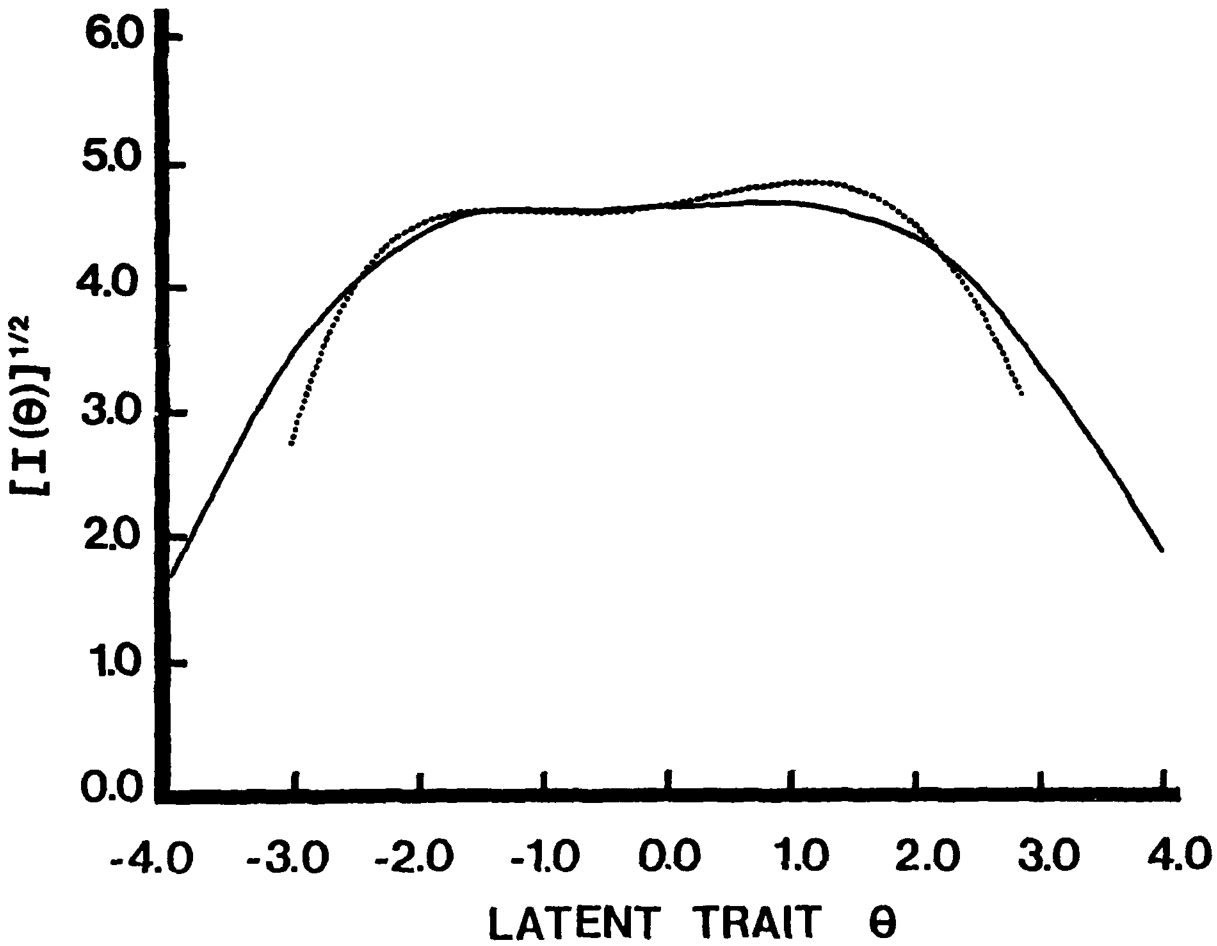

FIGURE 3-1 (Continued): Subtest 1, Polynomial of Degree 5, $[\underline{\theta}, \bar{\theta}] = [-3.0, 3.0]$.

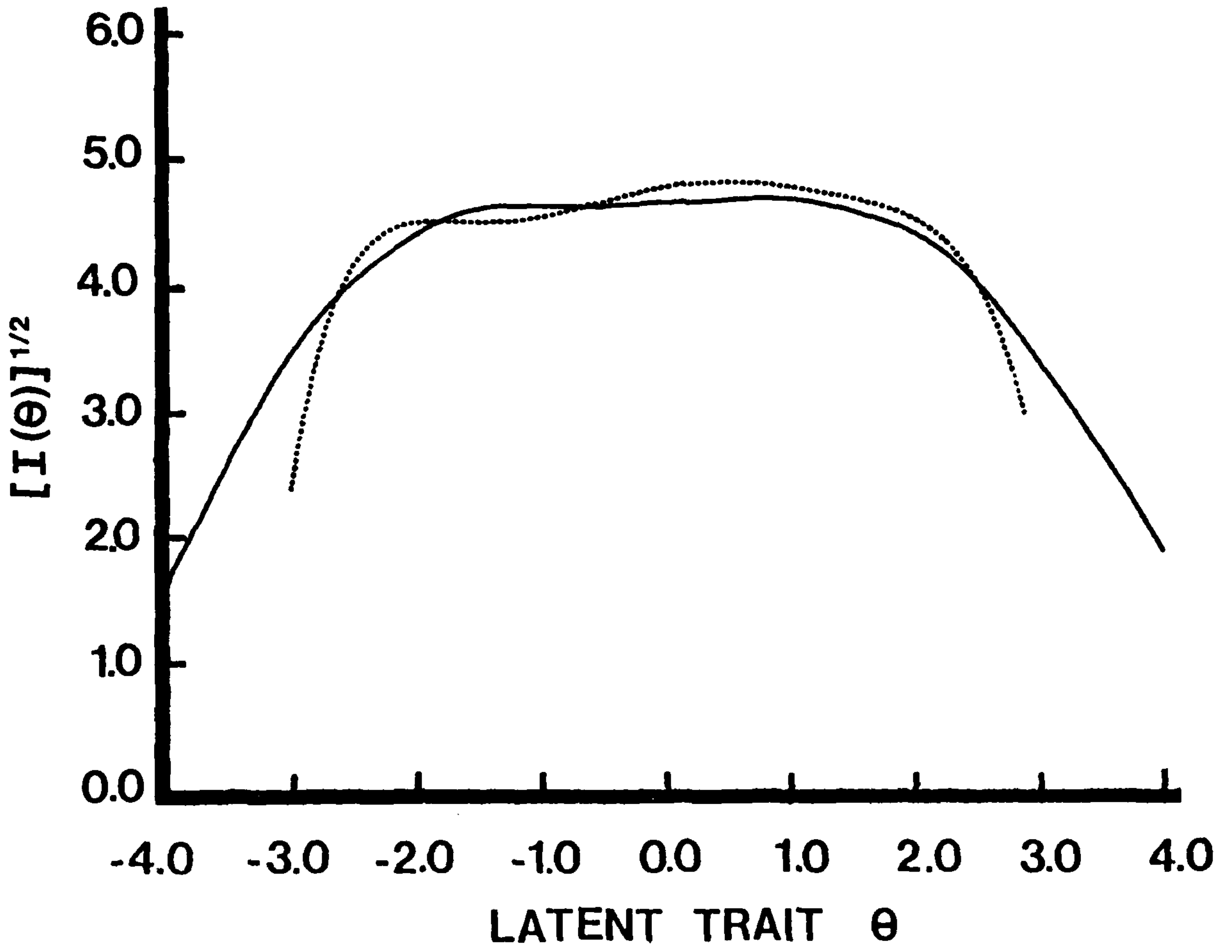

FIGURE 3-1 (Continued): Subtest 1, Polynomial of Degree 6, $[\underline{\theta}, \bar{\theta}] = [-3.0, 3.0]$.

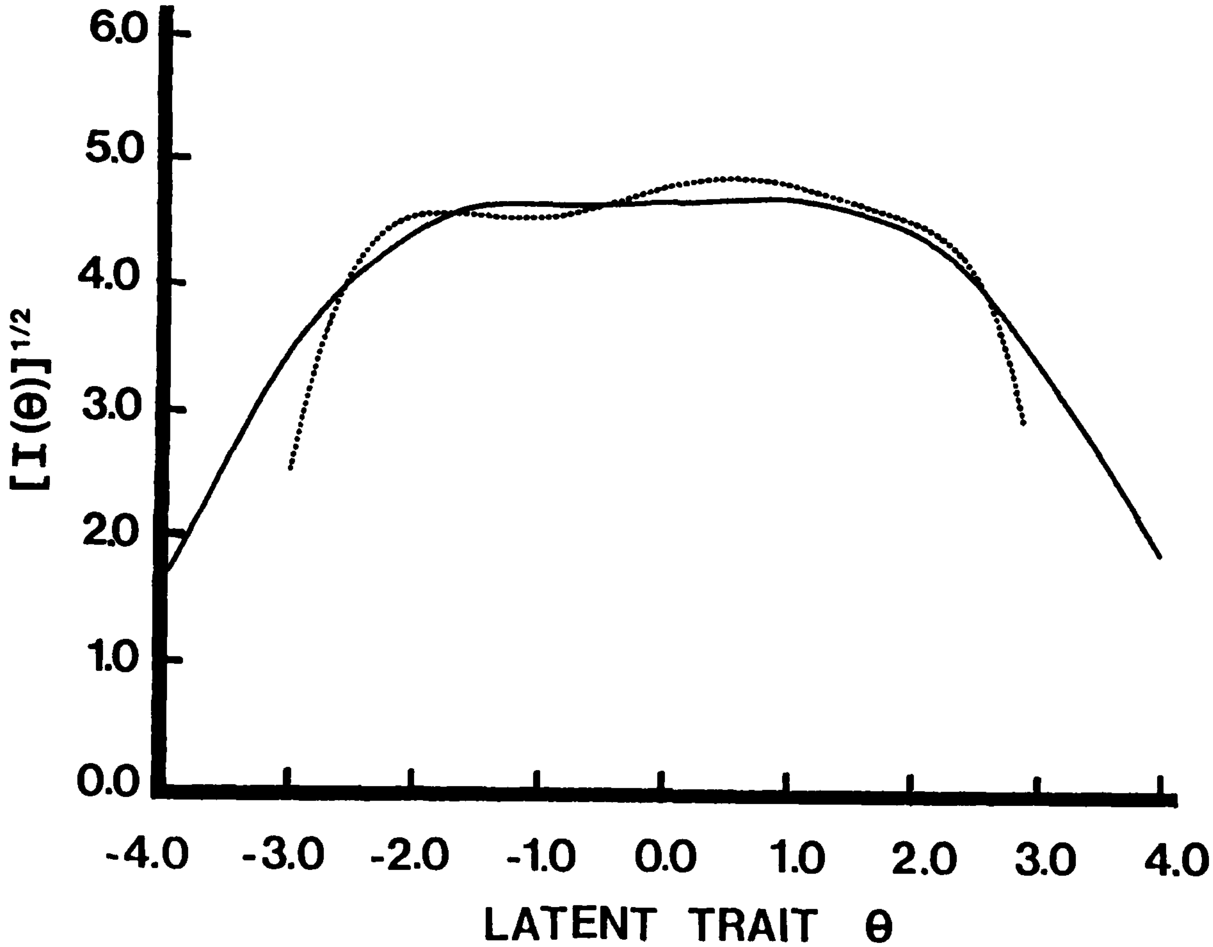

FIGURE 3-1 (Continued): Subtest 1, Polynomial of Degree 7, $[\underline{\theta}, \bar{\theta}]$ = [-3,0, 3.0].

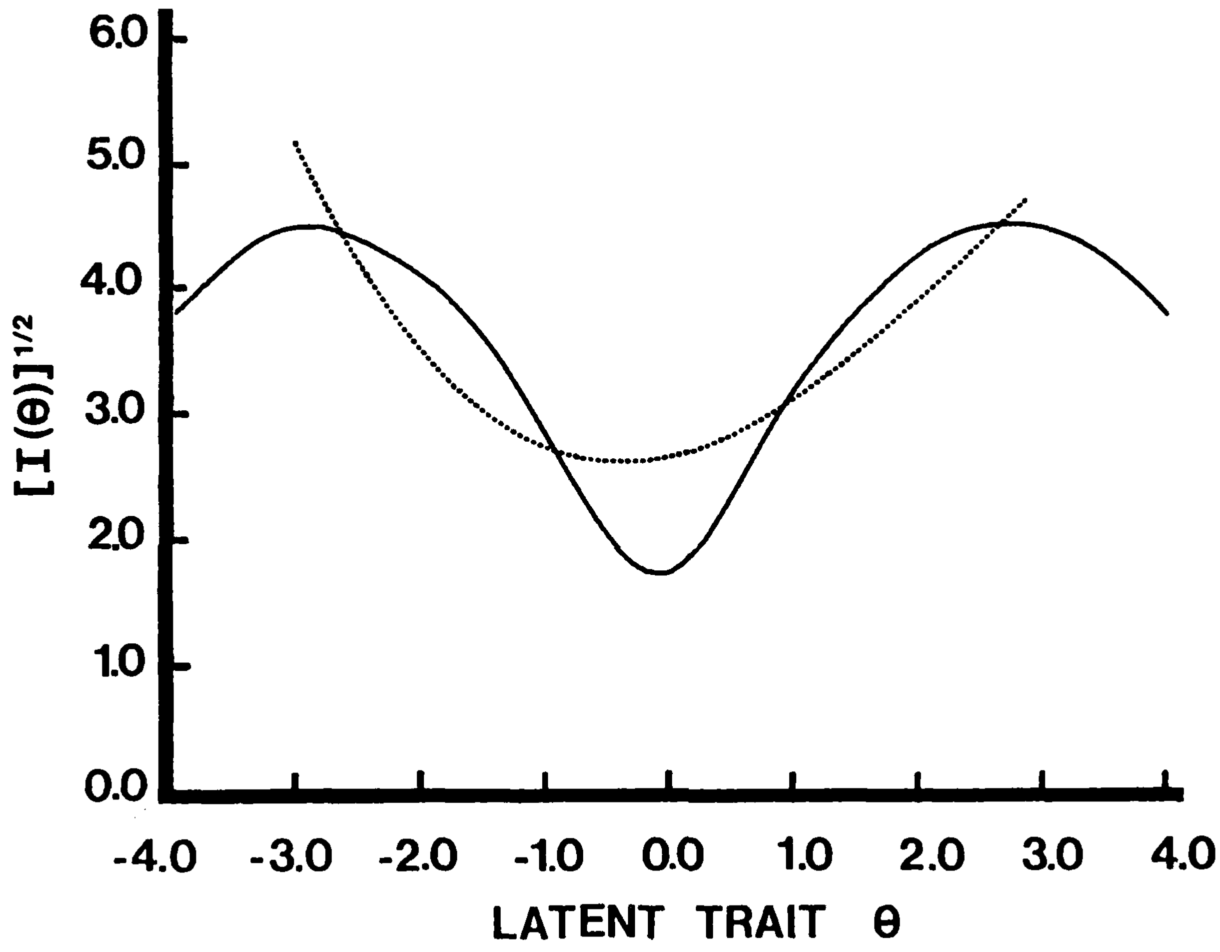

FIGURE 3-2

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) and the Polynomial of Degree 3 (Dotted Line), Which Was Fitted by the Method of Moments with [-3.0, 3.0] As the Interval of θ .

Subtest 2

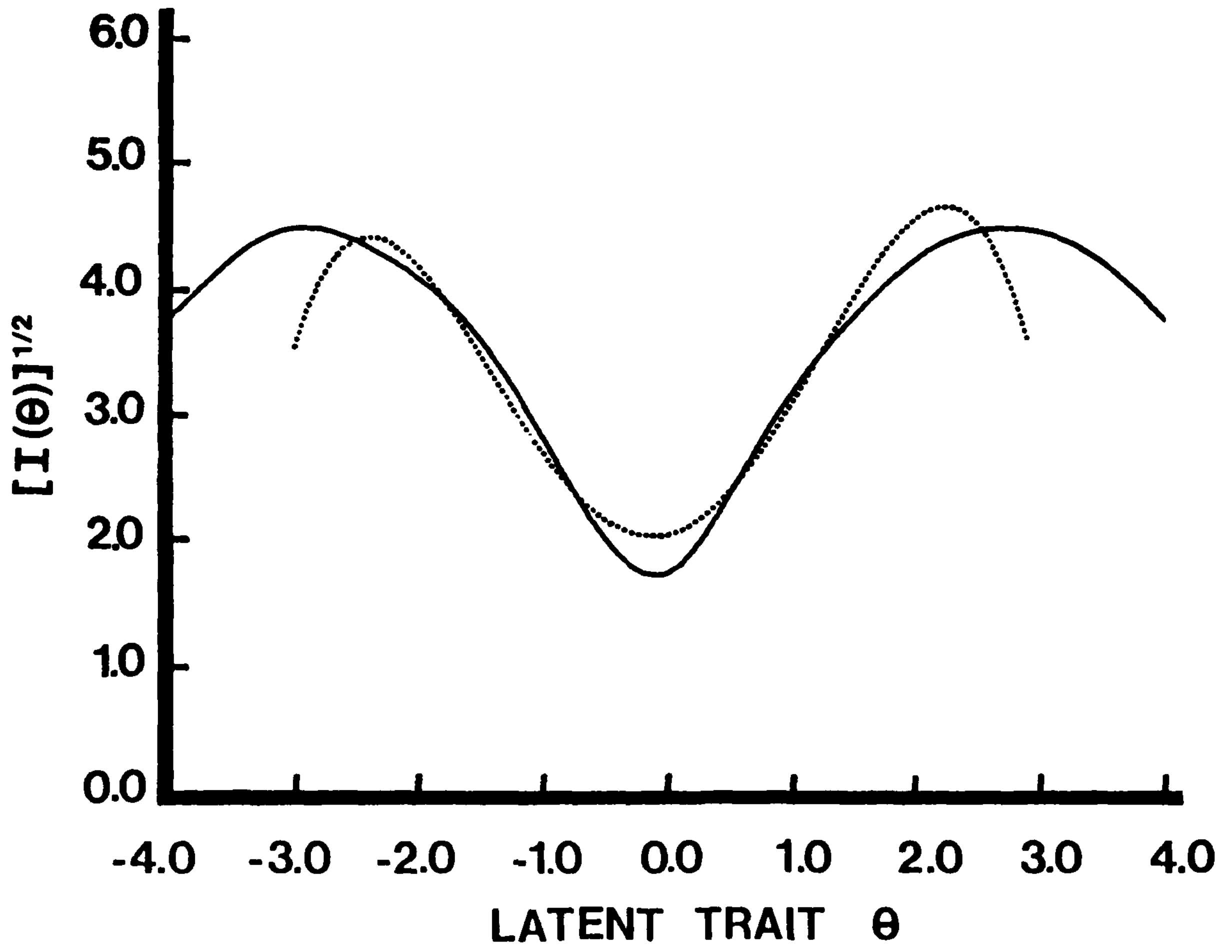

FIGURE 3-2 (Continued): Subtest 2, Polynomial of Degree 4, $[\underline{\theta}, \bar{\theta}] = [-3.0, 3.0]$.

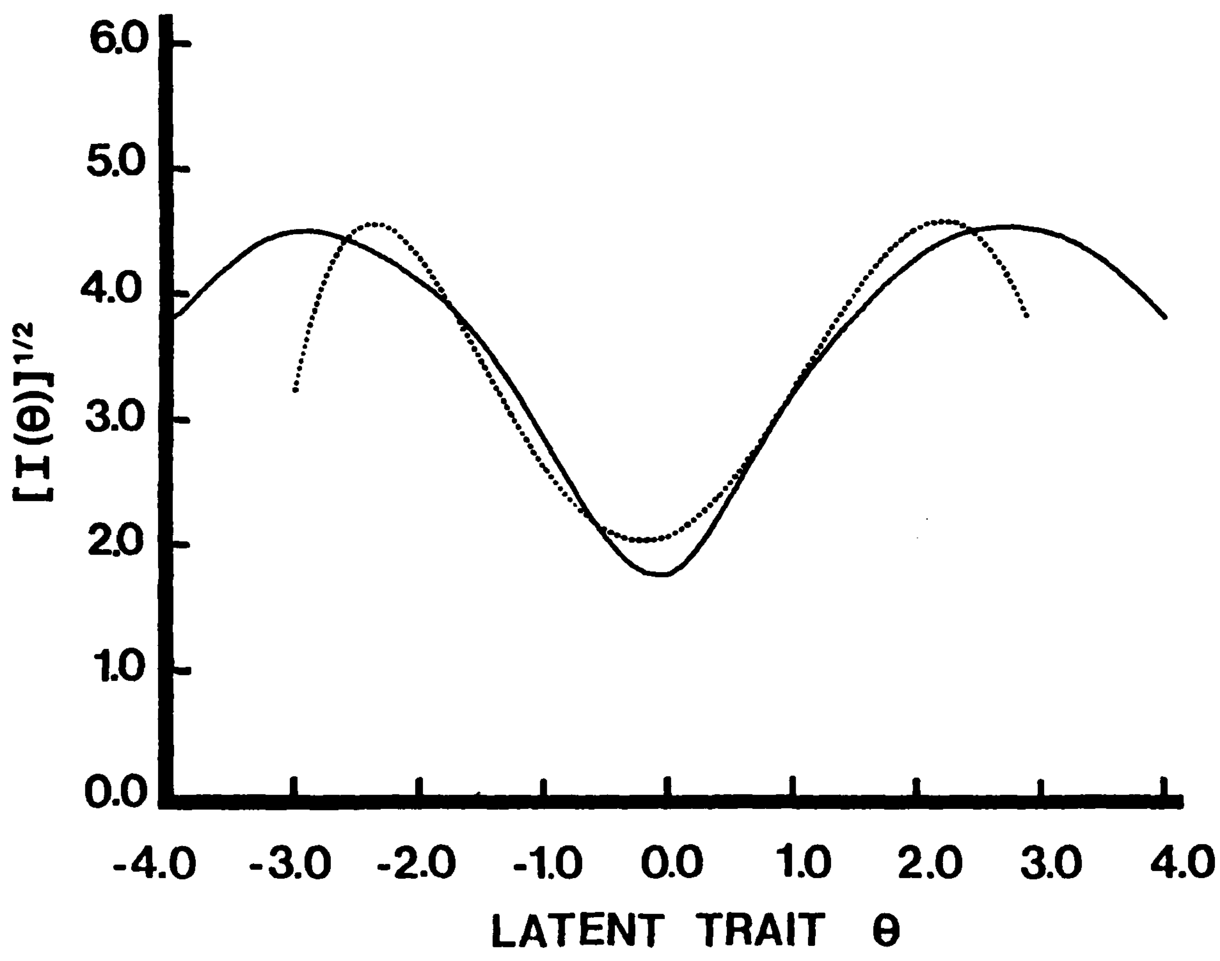

FIGURE 3-2 (Continued): Subtest 2, Polynomial of Degree 5, $[\underset{\sim}{\theta}, \tilde{\theta}]$ = [-3,0, 3,0].

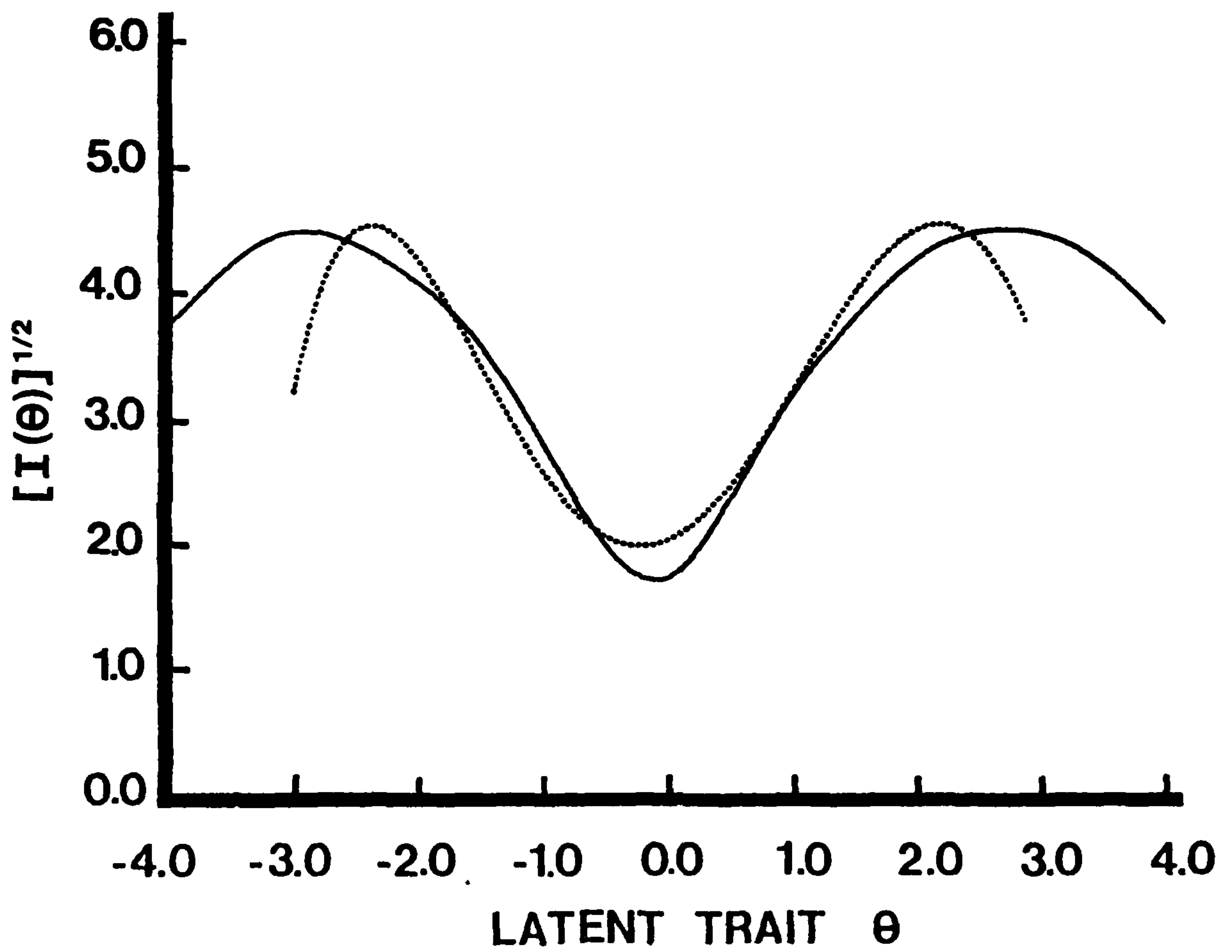

FIGURE 3-2 (Continued): Subtest 2, Polynomial of Degree 6, $[\underline{\theta}, \bar{\theta}] = [-3.0, 3.0]$.

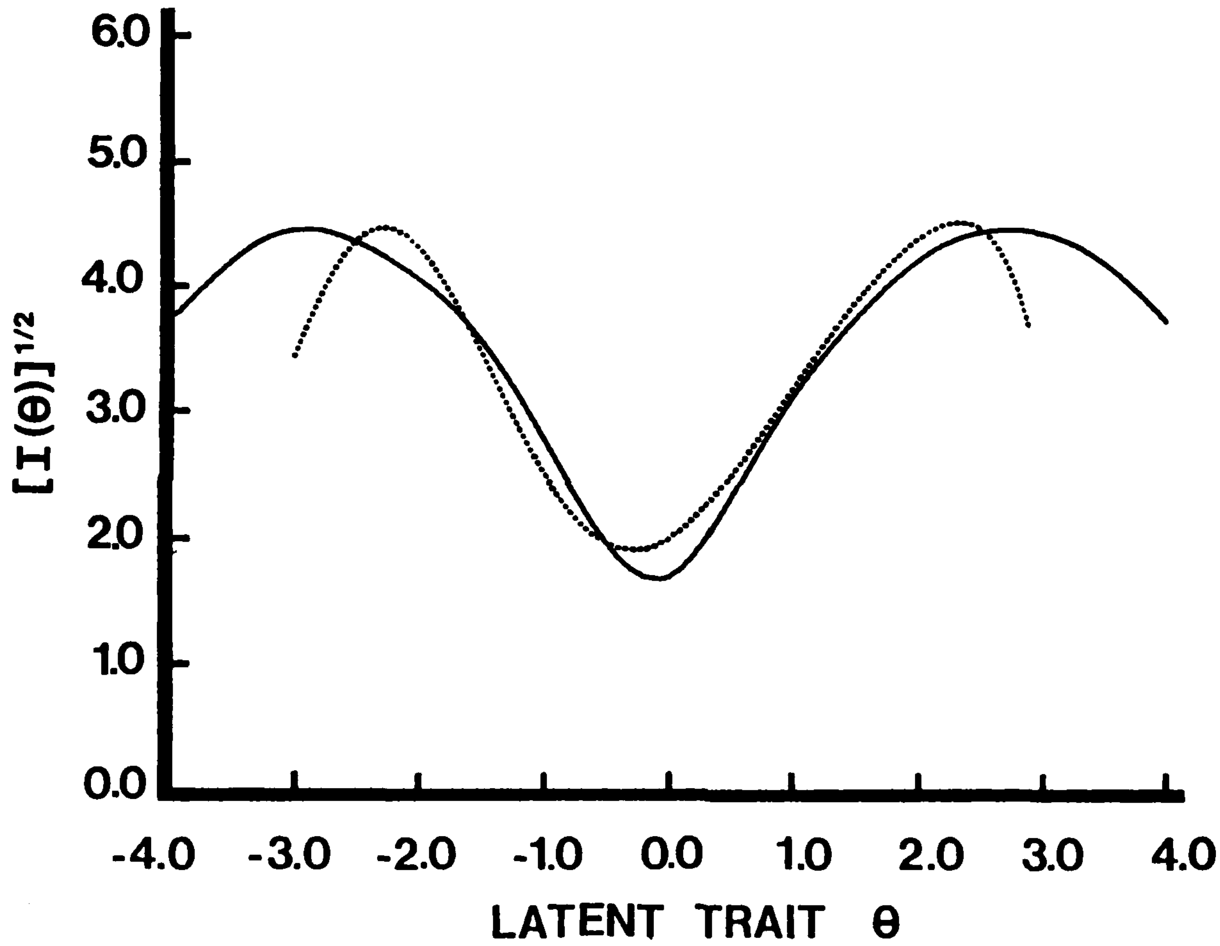

FIGURE 3-2 (Continued): Subtest 2, Polynomial of Degree 7, $[\underline{\theta}, \bar{\theta}]$ = [-3.0, 3.0]

origin, $\mu_r^{*\prime}$, which are given by

$$(3.9) \qquad \mu_r^{*\prime} = \int_{\underline{\theta}}^{\bar{\theta}} \theta^r [I(\theta)]^{1/2} \, d\theta \; , \qquad r=0,1,2,3,\ldots,m \; ,$$

were computed for each of the two subtests, where $m = 7$. Note that the 0-th moment is the area under the curve of $[I(\theta)]^{1/2}$ for the interval of θ , $[-3.0, 3.0]$, which is adjusted to unity. Since the midpoint of the interval, $[-3.0, 3.0]$, is the origin, these moments are also the moments about the midpoint, which we need in applying the method of moments. These moments turned out to be: 1.00000, 0.00768, 2.73116, -0.00547, 13.83270, -0.10637, 84.67312 and -0.92245 for Subtest 1, and: 1.00000, 0.04742, 3.54786, 0.10420, 19.44401, 0.38678, 123.79663 and 1.83934 for Subtest 2. The polynomials of degrees 3, 4, 5, 6 and 7 were obtained using the method of moments, and these five sets of coefficients are presented in Table 3-3 for Subtest 1, and in Table 3-4 for Subtest 2 (cf. Samejima and Livingston, 1979). These five polynomials are shown by dotted curves in Figures 3-1 and 3-2 for Subtests 1 and 2, respectively.

We can see in these ten graphs of Figures 3-1 and 3-2 that, although the polynomials fit fairly well to the square roots of the test information functions, there still is much to be desired, especially for extreme values of θ . For this reason, the same process was repeated for both Subtests 1 and 2, using a different interval for the method of moments, i.e., $\underline{\theta} = -4.0$ and

TABLE 3-3

Coefficients of the Polynomials of Degrees 3 through 7 Approximating $[I(\theta)]^{1/2}$, Which Were Obtained by the Method of Moments Using [-3.0, 3.0] and [-4.0, 4.0] As the Interval of θ, Respectively.

Subtest 1

		Interval	
		[-3.0, 3.0]	[-4.0, 4.0]
DGR. 3	0	4.90665	4.96268
	1	0.07842	0.00602
	2	-0.16475	-0.18690
	3	-0.01243	0.00021
DGR. 4	0	4.67066	4.73399
	1	0.07842	0.00602
	2	0.09745	-0.04398
	3	-0.01243	0.00021
	4	-0.03399	-0.01042
DGR. 5	0	4.67066	4.73399
	1	0.17323	0.05956
	2	0.09745	-0.04398
	3	-0.06159	-0.01541
	4	-0.03399	-0.01042
	5	0.00492	0.00088
DGR. 6	0	4.78242	4.72922
	1	0.17323	0.05956
	2	-0.16329	-0.03771
	3	-0.06159	-0.01541
	4	0.05290	-0.01160
	5	0.00492	0.00088
	6	-0.00708	0.00005
DGR. 7	0	4.78242	4.72922
	1	0.26677	0.10599
	2	-0.16329	-0.03771
	3	-0.15513	-0.04152
	4	0.05290	-0.01160
	5	0.02778	0.00447
	6	-0.00708	0.00005
	7	-0.00157	-0.00014

TABLE 3-4

Coefficients of the Polynomials of Degrees 3 through 7 Approximating $[I(\theta)]^{1/2}$, Which Were Obtained by the Method of Moments Using [-3.0, 3.0] and [-4.0, 4.0] As the Interval of θ, Respectively.

Subtest 2

		Interval	
		[-3.0, 3.0]	[-4.0, 4.0]
DGR. 3	0	2.63641	3.02995
	1	0.22214	0.10837
	2	0.25995	0.10841
	3	-0.03114	-0.00924
DGR. 4	0	2.02466	2.27454
	1	0.22214	0.10837
	2	0.93968	0.58054
	3	-0.03114	-0.00924
	4	-0.08811	-0.03443
DGR. 5	0	2.02466	2.27454
	1	0.41951	0.24669
	2	0.93968	0.58054
	3	-0.13348	-0.04958
	4	-0.08811	-0.03443
	5	0.01023	0.00227
DGR. 6	0	2.02136	2.14813
	1	0.41951	0.24669
	2	0.94740	0.74646
	3	-0.13348	-0.04958
	4	-0.09071	-0.06554
	5	0.01023	0.00227
	6	0.00021	0.00143
DGR. 7	0	2.02136	2.14813
	1	0.60587	0.37926
	2	0.94740	0.74646
	3	-0.31984	-0.12415
	4	-0.09071	-0.06554
	5	0.05579	0.01252
	6	0.00021	0.00143
	7	-0.00313	-0.00040

$\bar{\theta} = 4.0$. The new set of eight moments about the origin, which were computed through (3.9), proved to be: 1.00000, 0.01082, 4.26091, 0.10885, 35.49275, 1.61607, 367.31471 and 24.05220 for Subtest 1, and: 1.00000, 0.02913, 6.01702, 0.03999, 56.94637, -0.09788, 633.40916 and -3.04930 for Subtest 2. The coefficients of the resultant five polynomials are also presented in Table 3-3 for Subtest 1, and in Table 3-4 for Subtest 2. Figures 3-3 and 3-4 present the new polynomials of degree 3, 4, 5, 6 and 7 by dotted curves, together with the square root of the test information function, which is shown by a solid curve, for Subtests 1 and 2, respectively. We can see a substantial improvement in the fit of polynomials for both subtests, and, especially for Subtest 1, the polynomial whose degree is as low as 4 already provides us with an excellent fit.

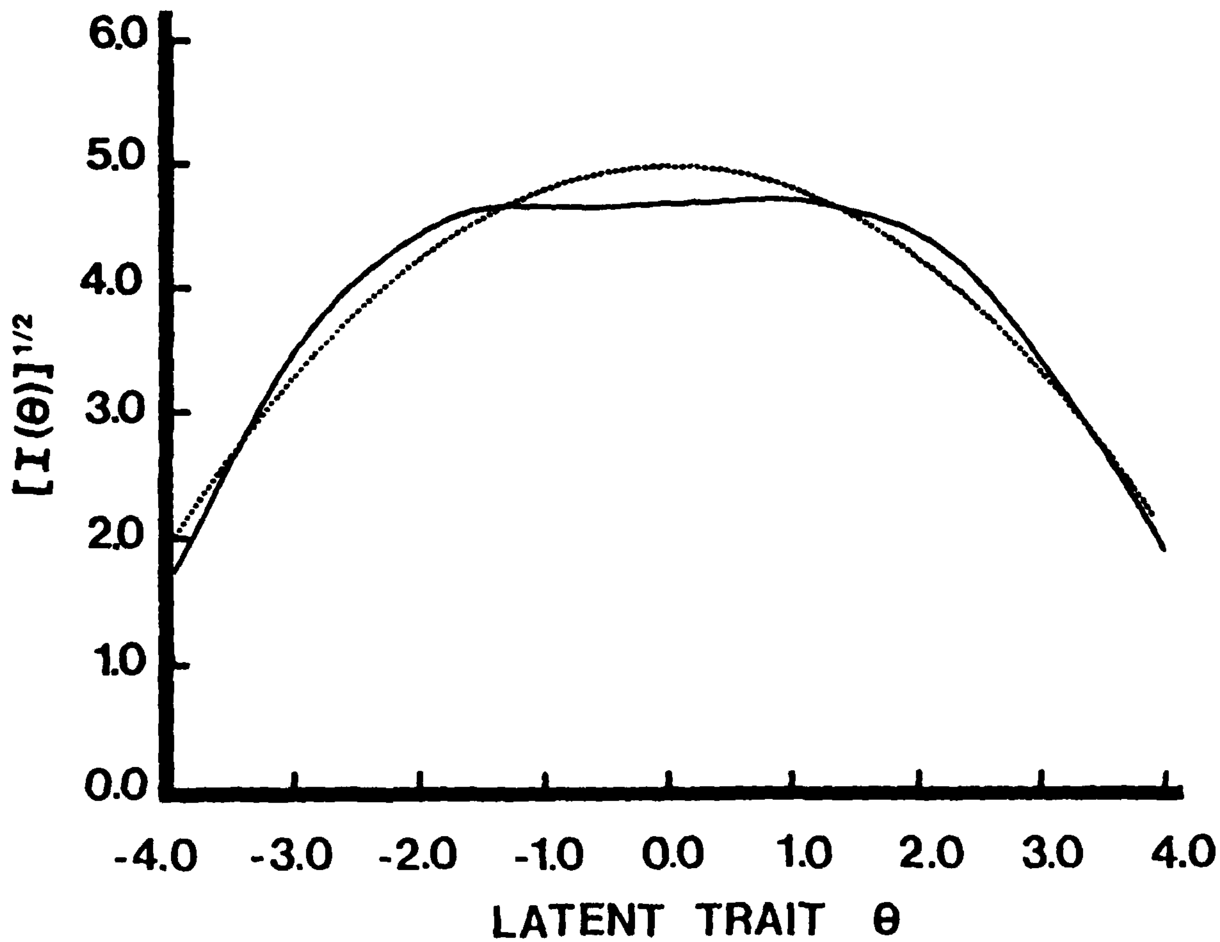

FIGURE 3-3

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) and the Polynomial of Degree 3 (Dotted Line), Which Was Fitted by the Method of Moments with $[-4.0, 4.0]$ As the Interval of θ .

Subtest 1

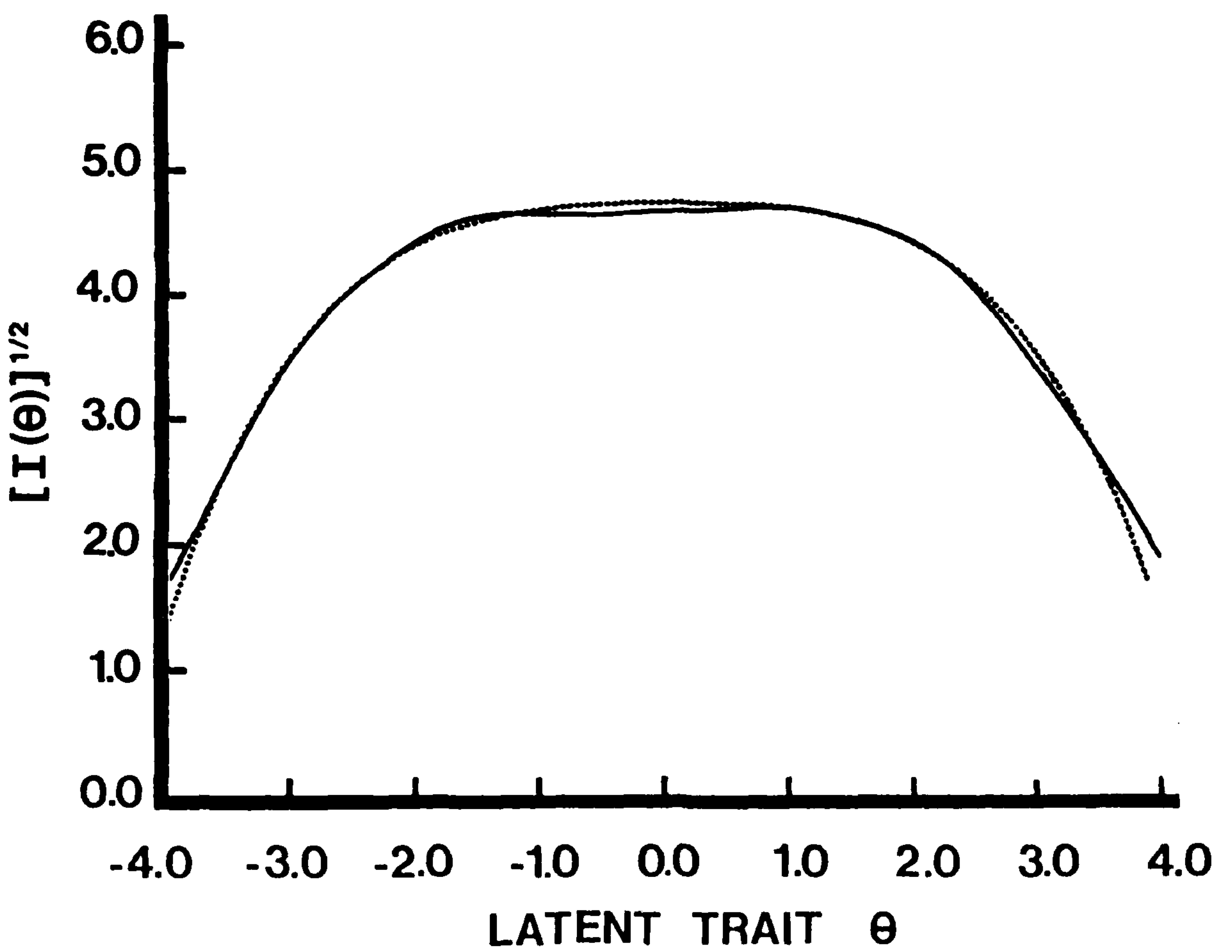

FIGURE 3-3 (Continued): Subtest 1, Polynomial of Degree 4, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

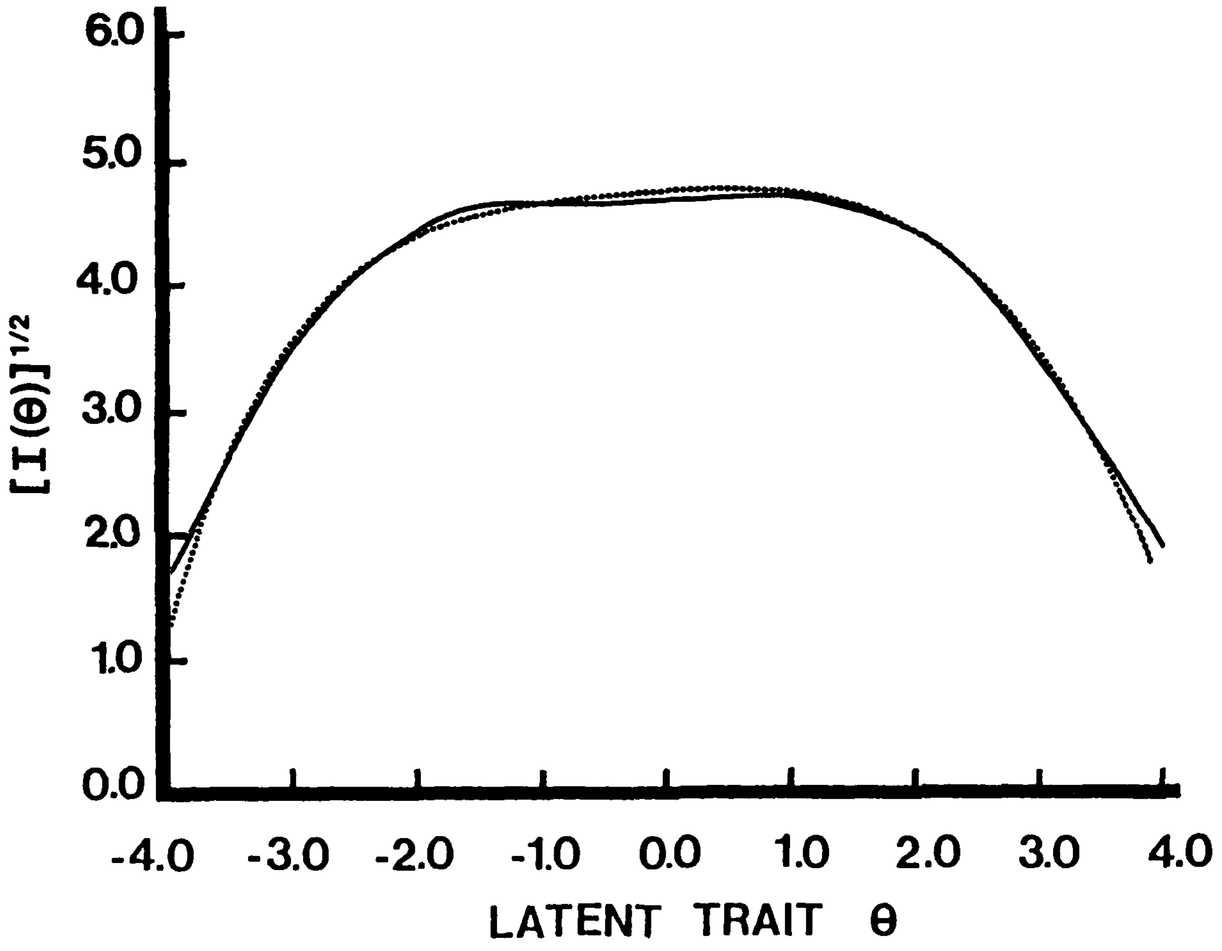

FIGURE 3-3 (Continued): Subtest 1, Polynomial of Degree 5, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

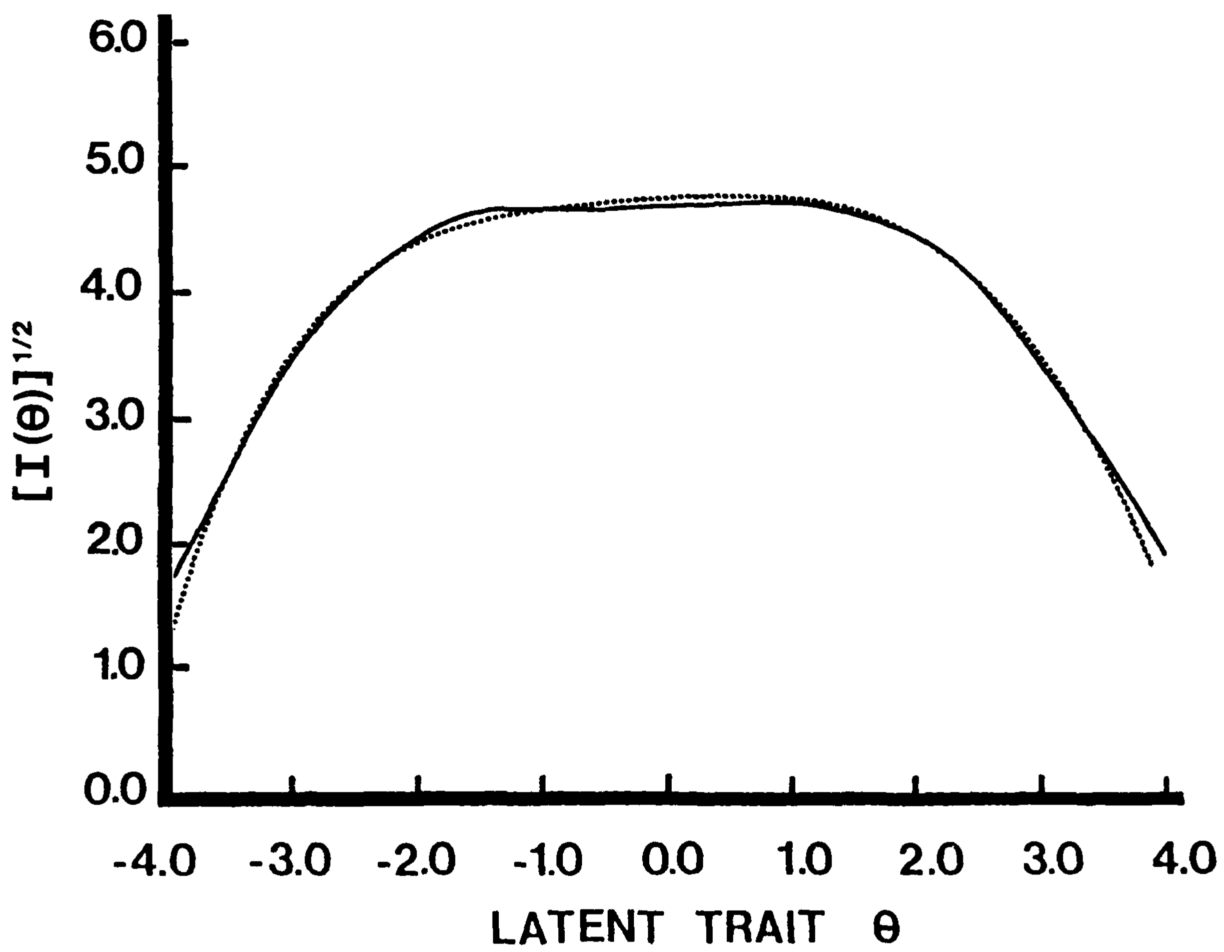

FIGURE 3-3 (Continued): Subtest 1, Polynomial of Degree 6, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

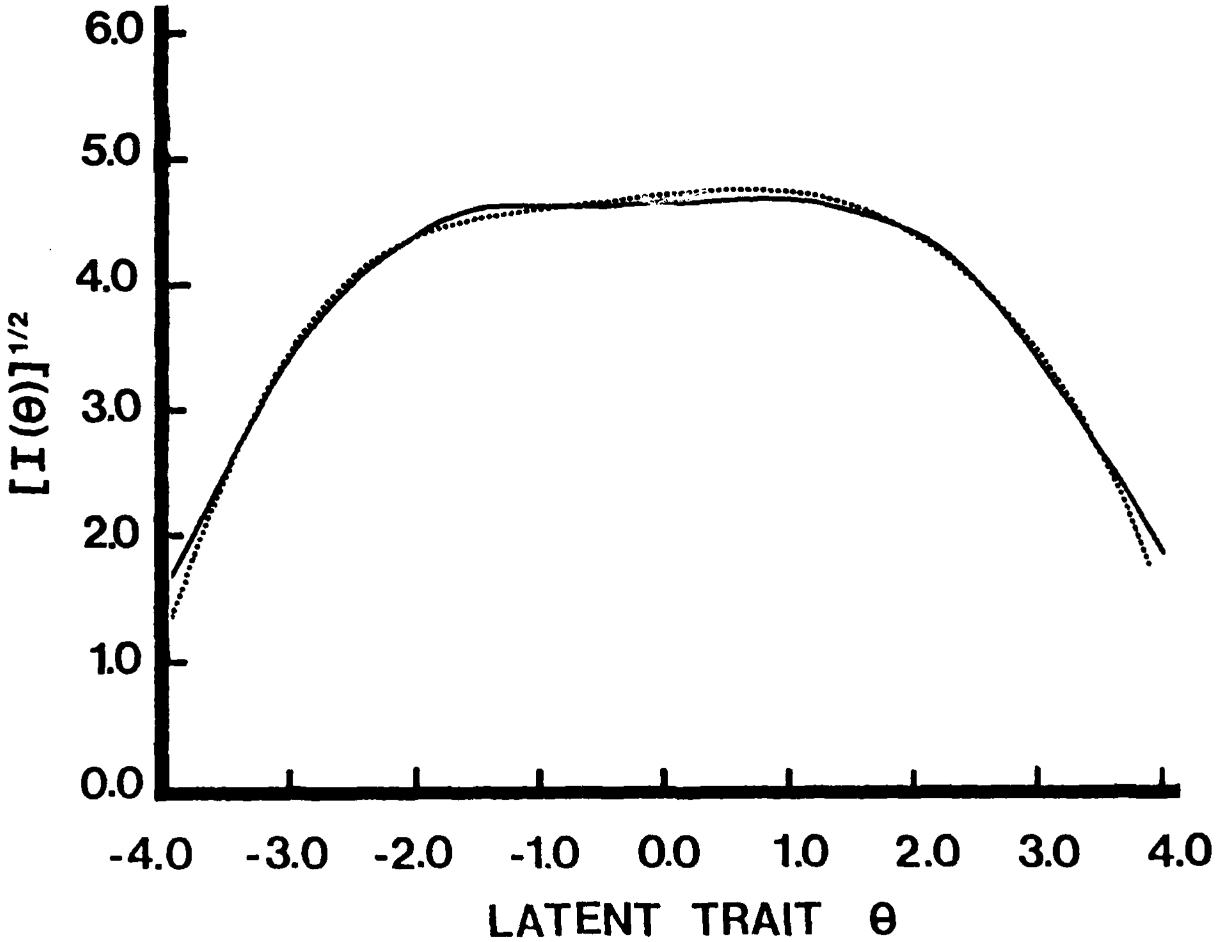

FIGURE 3-3 (Continued): Subtest 1, Polynomial of Degree 7, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

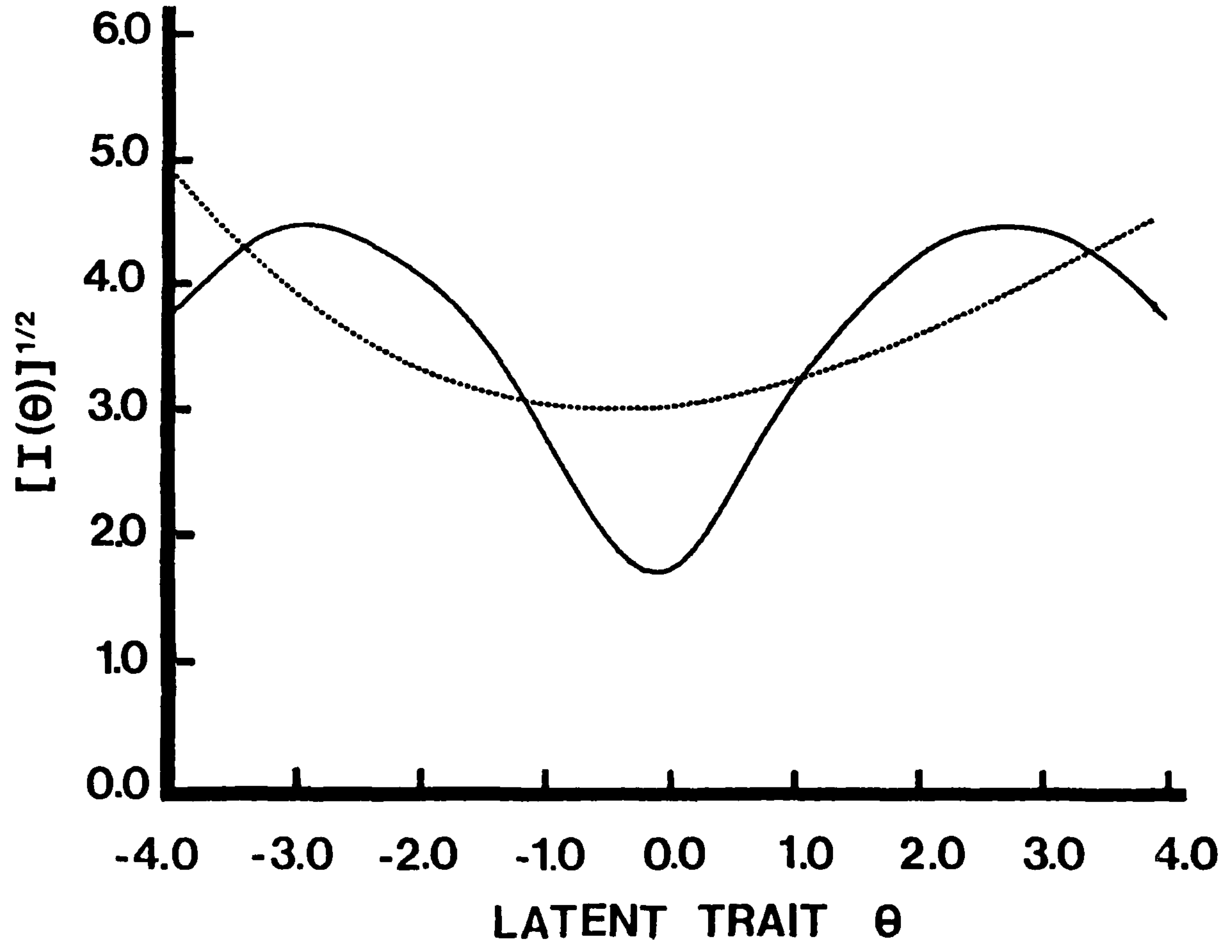

FIGURE 3-4

Square Root of the Test Information Function, $[I(\theta)]^{1/2}$, (Solid Line) and the Polynomial of Degree 3 (Dotted Line), Which Was Fitted by the Method of Moments with [-4.0, 4.0] As the Interval of θ .

Subtest 2

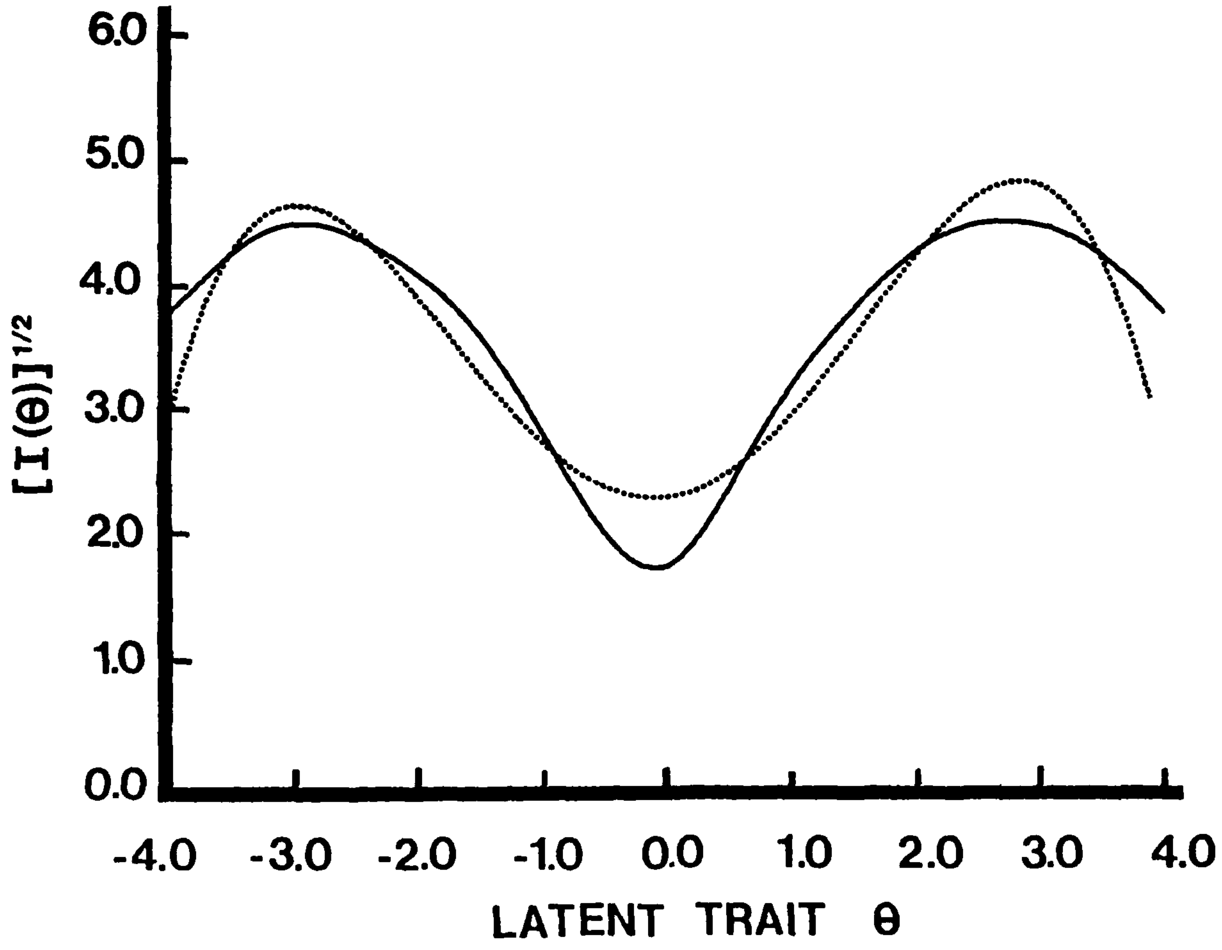

FIGURE 3-4 (Continued): Subtest 2, Polynomial of Degree 4, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

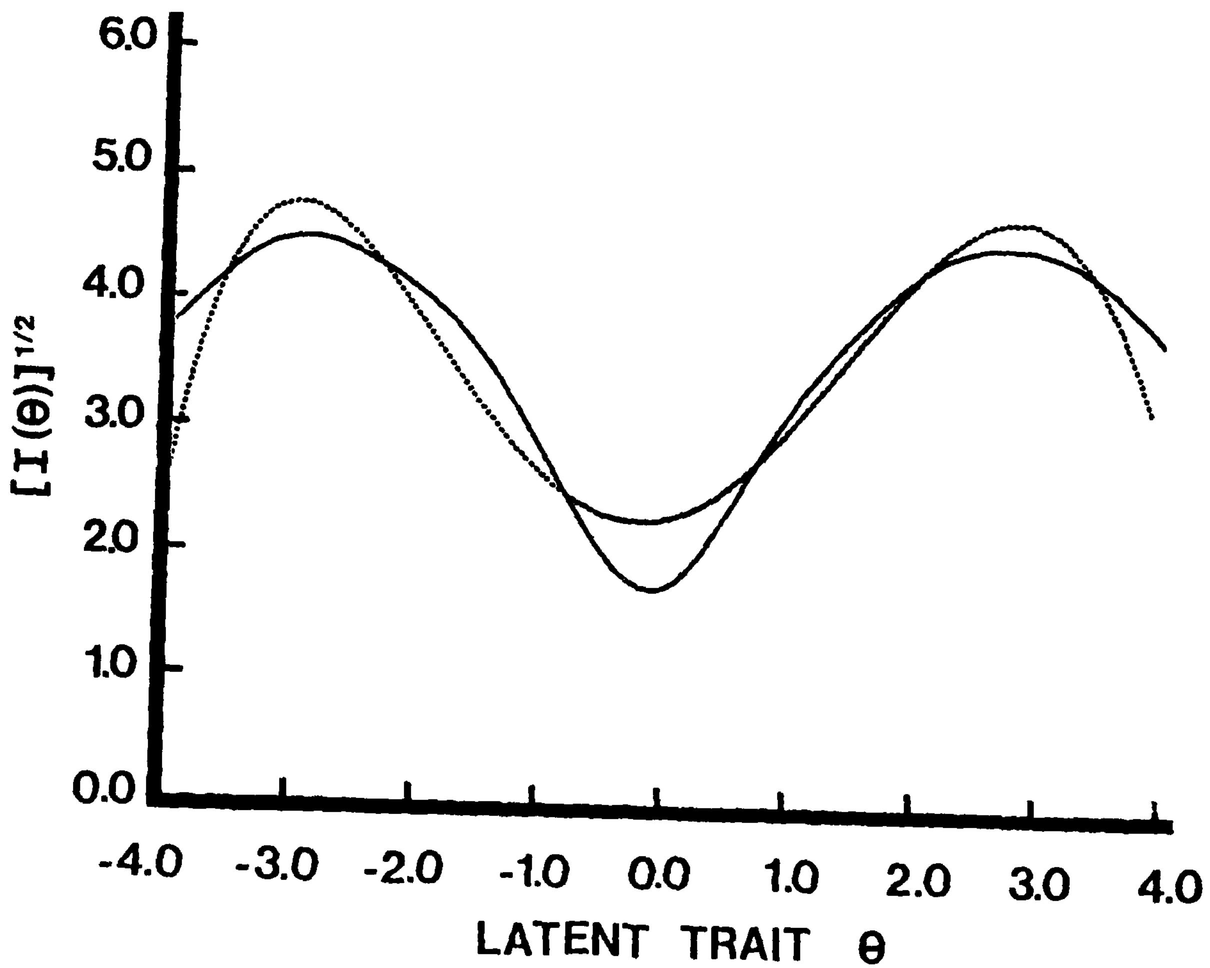

FIGURE 3-4 (Continued): Subtest 2, Polynomial of Degree 5, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

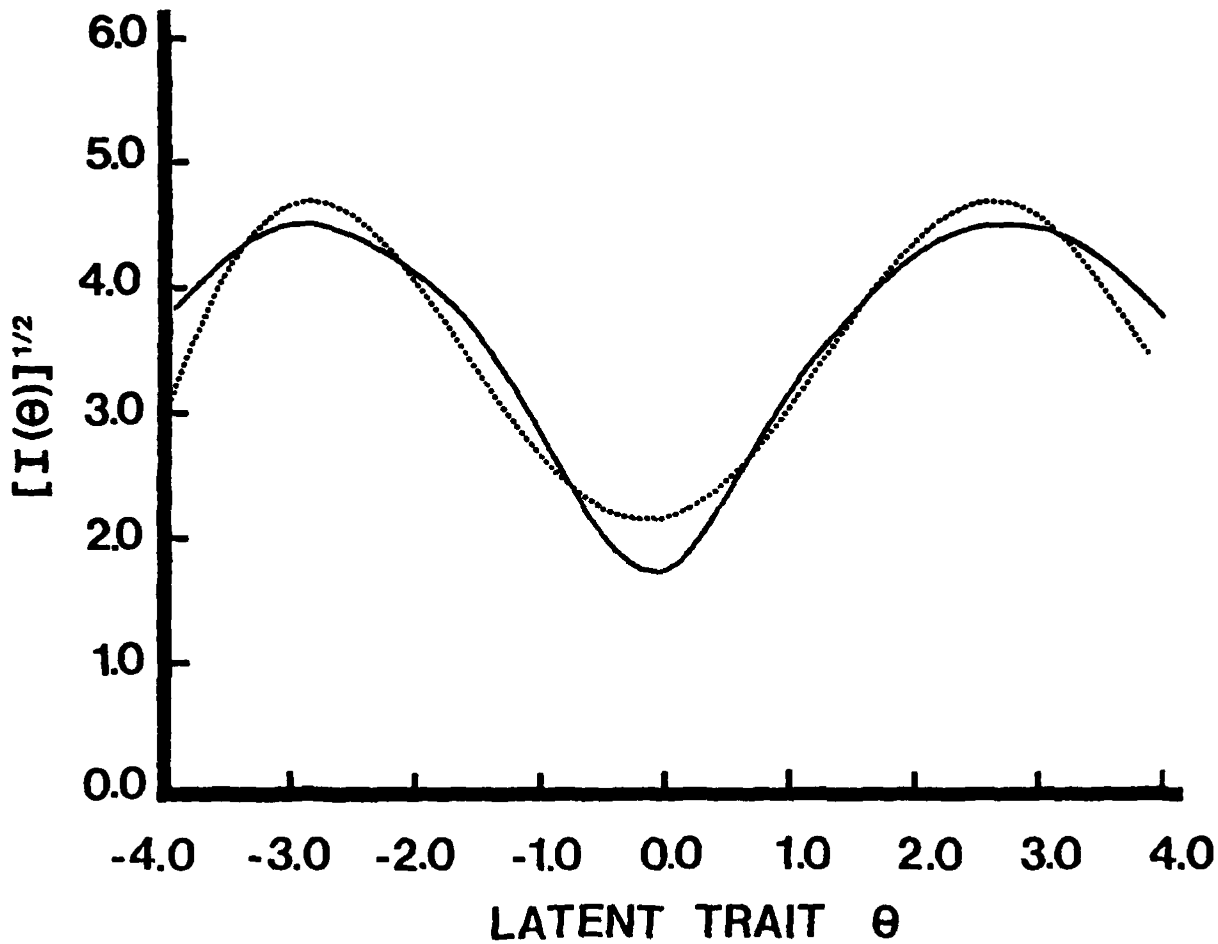

FIGURE 3-4 (Continued): Subtest 2, Polynomial of Degree 6, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$.

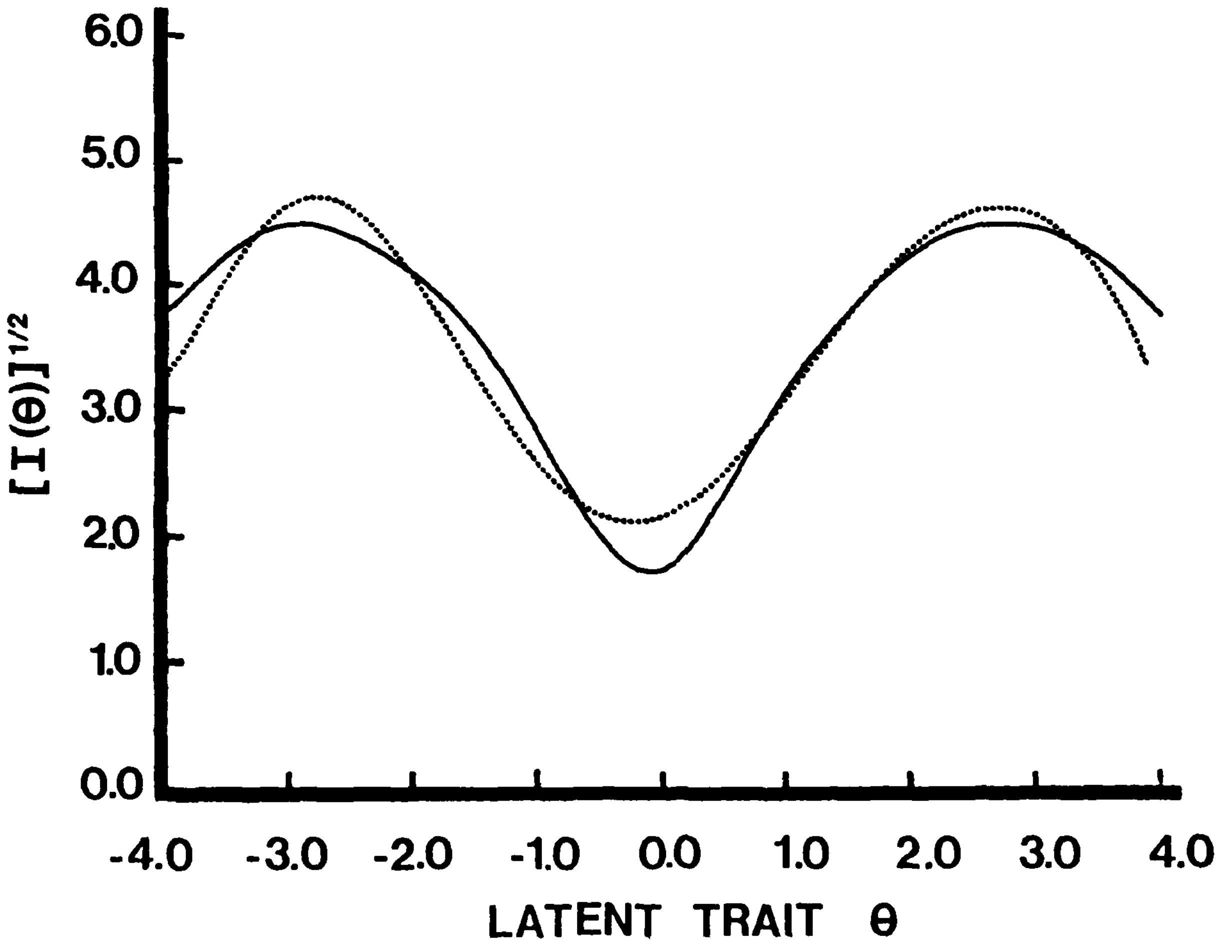

FIGURE 3-4 (Continued): Subtest 2, Polynomial of Degree 7, $[\underline{\theta}, \bar{\theta}] = [-4.0, 4.0]$

IV Basic Data for Estimating the Operating Characteristics

We must administer both Old Test, whose test information function needs not to be constant, and the set of new items, whose operating characteristics are to be estimated, to, say, several hundred examinees, whom we sampled from an appropriate population, as is the case in the previous studies, in which we used an Old Test whose test information function is constant. Let N denote the number of examinees. It is required that the "known" test items of the Old Test follow a model, or models, which provides us with a unique maximum likelihood estimate for every possible response pattern (cf. Samejima, 1969, 1972).

Next, we must obtain the maximum likelihood estimate, $\hat{\theta}$, of ability θ for every individual examinee from his response pattern V on the Old Test of n items. When there exists a simple sufficient statistic for the response pattern, as in the logistic model on the dichotomous response level, this process is relatively simple and straight forward. That is to say, in the logistic model whose item characteristic function, $P_g(\theta)$, or the operating characteristic for $x_g=1$ on the dichotomous response level, is given by

$$P_g(\theta) = [1 + \exp\{-1.7\ a_g(\theta-b_g)\}]^{-1} \quad , \tag{4.1}$$

where a_g and b_g are the discrimination and difficulty parameters, respectively, the maximum likelihood estimate is the solution of θ

to the equation

(4.2) $$t(V) = \sum_{g=1}^{n} a_g P_g(\theta) \quad ,$$

where $t(V)$ is a simple sufficient statistic for the response pattern V which is given by

(4.3) $$t(V) = \sum_{x_g \varepsilon V} a_g x_g$$

(cf. Birnbaum, 1968). When there exists no sufficient statistic for the response pattern, as is the case in most situations, the maximum likelihood estimate must be obtained through a more complicated numerical process, using $[\sum_{g=1}^{n} m_g + n]$ basic functions (Samejima, 1969, 1972), $A_{x_g}(\theta)$, which is defined by

(4.4) $$A_{x_g}(\theta) = \frac{\partial}{\partial\theta} P_{x_g}(\theta)$$

for each graded item response x_g . Thus the maximum likelihood estimate is the solution to the equation,

(4.5) $$\sum_{x_g \varepsilon V} A_{x_g}(\theta) = 0 \quad ,$$

which can be obtained by the aid of an electronic computer using Newton-Raphson Method.

The third step is to compute the test information function, $I(\theta)$, of the Old Test through (2.2), (2.3) and (2.8), and, once it has been done, its square root, $[I(\theta)]^{1/2}$, must be computed.

Then we calculate the moments of $[I(\theta)]^{1/2}$ about the midpoint of the interval, $[\underline{\theta}, \bar{\theta}]$, and apply the method of moments to obtain the polynomial which approximates $[I(\theta)]^{1/2}$. In so doing, it is important to adjust the endpoints of the interval, $\underline{\theta}$ and $\bar{\theta}$, and the degree of the polynomial m , as was illustrated in the preceding chapter, in order to obtain a good approximation. Thus the $(m+1)$ coefficients, α_k $(k=0,1,2,\ldots,m)$, in (3.4) have been obtained for the Old Test.

After this has been done, set the desired amount of constant test information, C^2 , for the second test information function, $I^*(\tau)$, which is to be used after the transformation of θ to τ . Since the normal approximation to the conditional distribution of $\hat{\tau}$, given τ , plays an essential role in the estimation methods, this constant amount of test information must be substantially large.

Next, we must obtain the coefficients α_k^* $(k=0,1,2,\ldots m,m+1)$ in the transformation of θ to τ , which is given by (3.5). First, determine the value of τ corresponding to the origin of θ , and use this as d in (3.5). If we wish to keep the position of the origin unchanged, then set $d = 0$. Using these two values of C (>0) and d thus obtained, and the coefficients α_k's of the polynomial approximating $[I(\theta)]^{1/2}$, obtain the coefficients, α_k^* , of the polynomial given by (3.5) from (3.6).

The final step is to obtain the maximum likelihood estimate $\hat{\tau}$, of the transformed latent trait τ , on the Old Test, for each

of the N examinees. We may do this through the equation

$$\hat{\tau} \doteq \sum_{k=0}^{m+1} \alpha_k^* \hat{\theta}^k , \tag{4.6}$$

where $\hat{\theta}$ is the maximum likelihood estimate of θ on the Old Test for each individual examinee, which was obtained earlier. This set of the maximum likelihood estimates $\hat{\tau}$ for the total group of N examinees is the basic data for each estimation process of the operating characteristics of the graded item responses, which is to be presented in a later chapter.

For the purpose of illustration, Figures 4-1 and 4-2 present the relative frequency distributions of $\hat{\theta}$ and $\hat{\tau}$ for the five hundred hypothetical subjects, respectively, which were obtained through Subtest 1. This subtest consists of twenty-five graded test items which follow the normal ogive model, with the discrimination and difficulty parameters shown in Tables 3-1 and 3-2, respectively, as was introduced in the preceding chapter. The values of $\hat{\theta}$ were obtained by using the basic function defined by (4.4) for each item score x_g , and as the solution to the equation (4.5). The transformation of $\hat{\theta}$ to $\hat{\tau}$ was made through (4.6) with $m = 7$, in which the coefficients, α_k^*'s , were based on the coefficients α_k's obtained by the method of moments with $\underline{\theta} = -4.0$ and $\bar{\theta} = 4.0$, and $C = 4.5$. These coefficients, α_k^*'s , are shown in Table 3-3. As we can see in these two figures, the frequency distribution of $\hat{\tau}$ turned out to be more rectangular

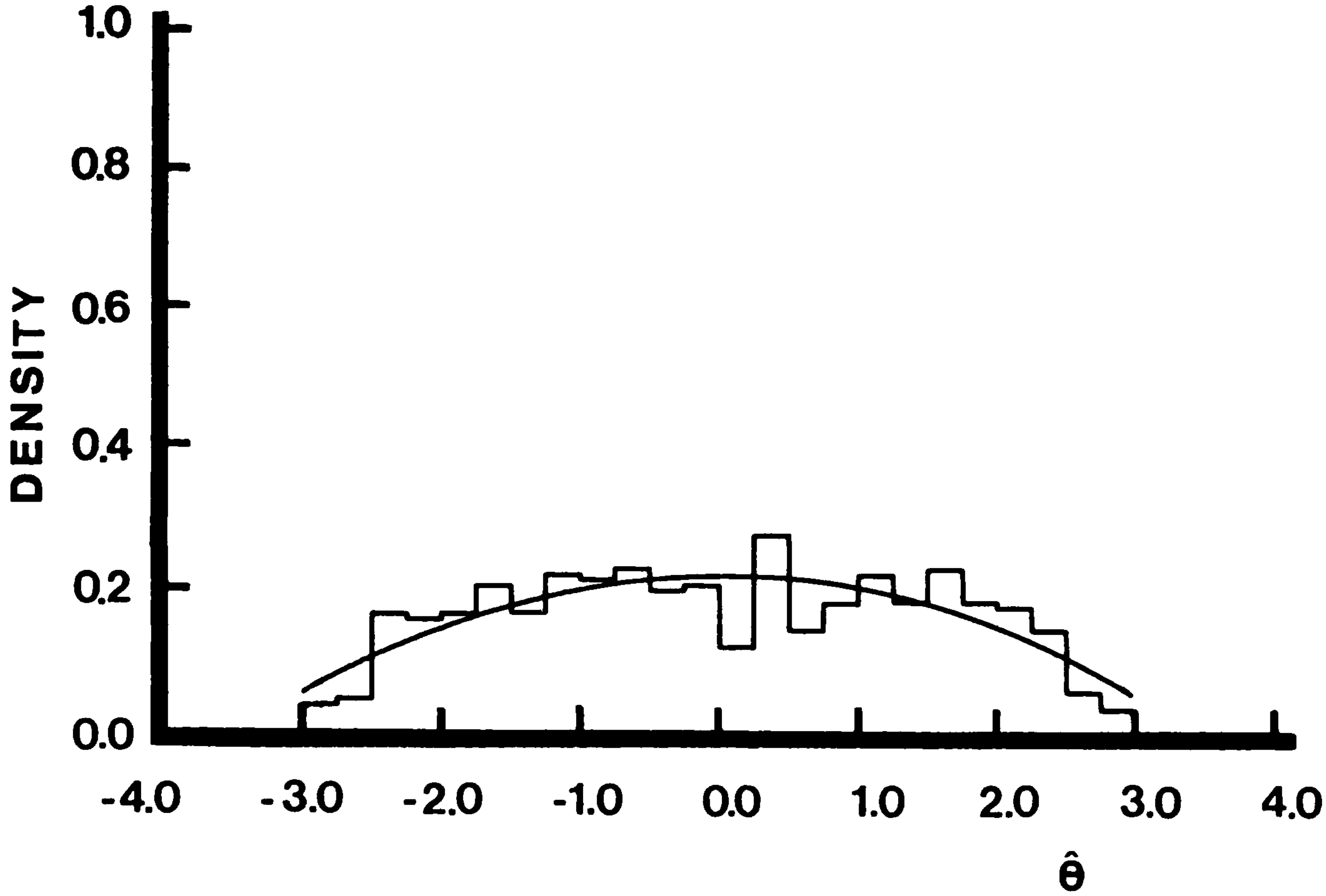

FIGURE 4-1

Relative Frequency Distribution of $\hat{\theta}$, Which Was Obtained for the Five Hundred Hypothetical Examinees on Subtest 1, with 0.25 as the Subinterval Width, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate the Density Function of $\hat{\theta}$.

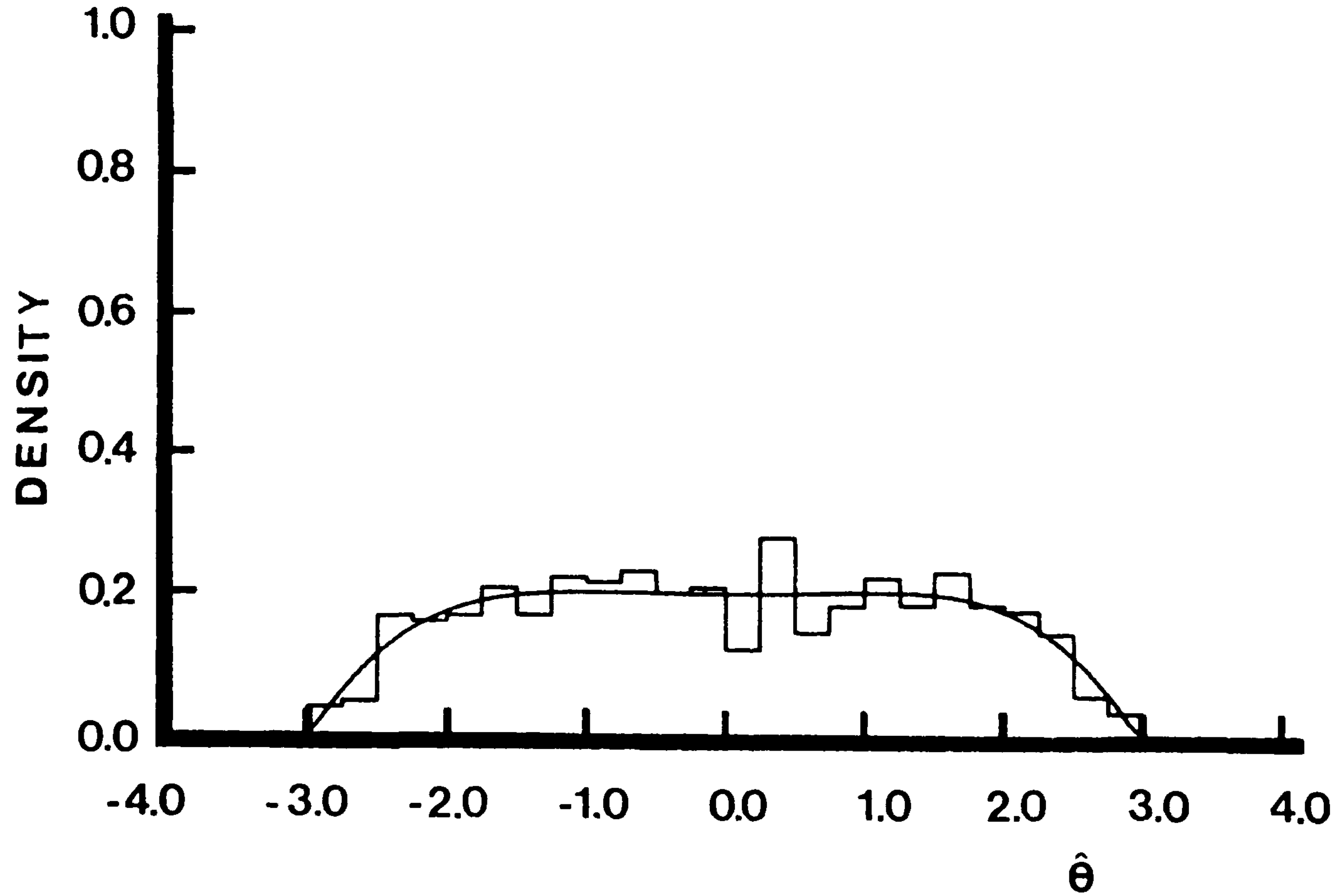

FIGURE 4-1 (Continued): Subtest 1, $\hat{\theta}$, Polynomial of Degree 4.

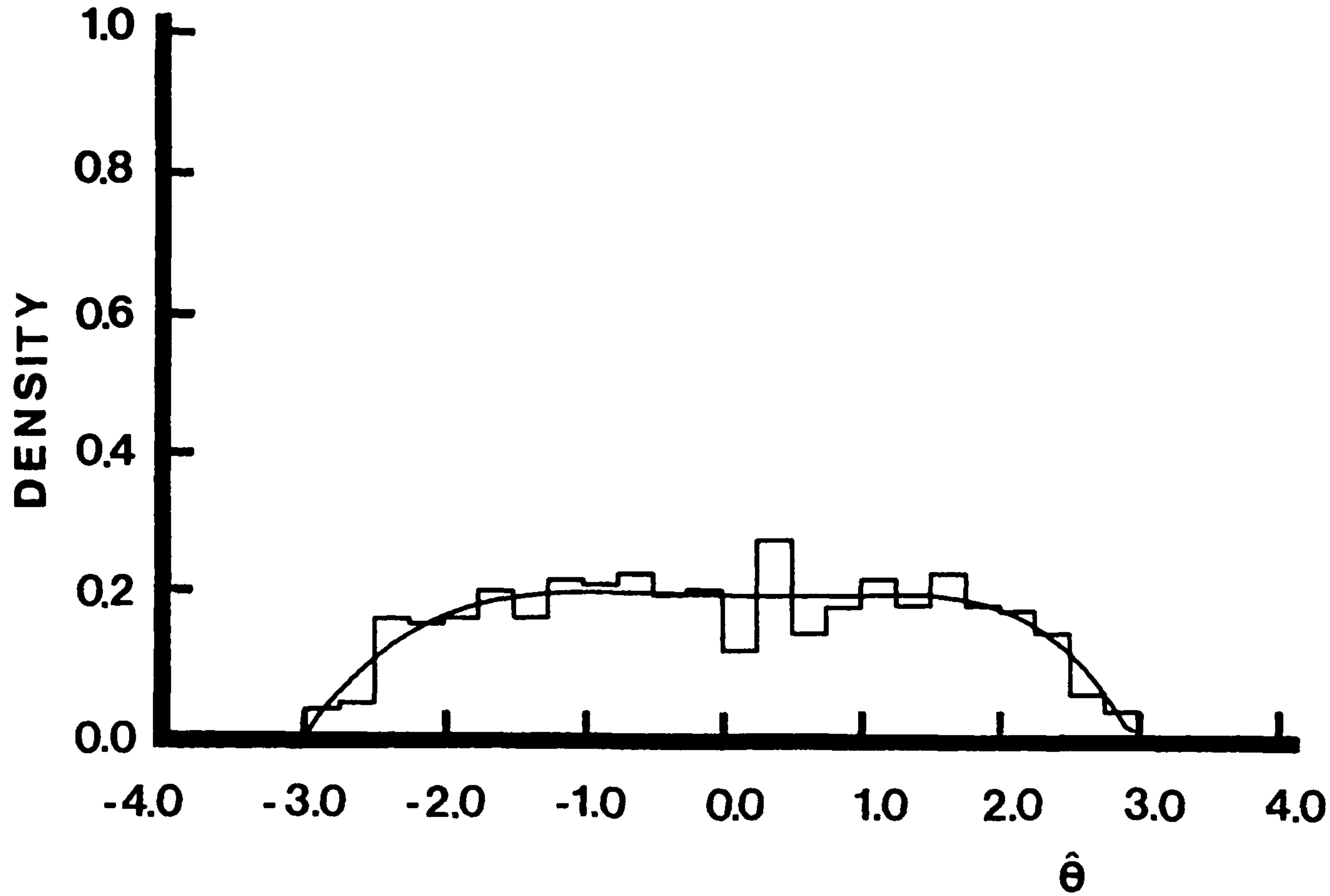

FIGURE 4-1 (Continued): Subtest 1, $\hat{\theta}$, Polynomial of Degree 5.

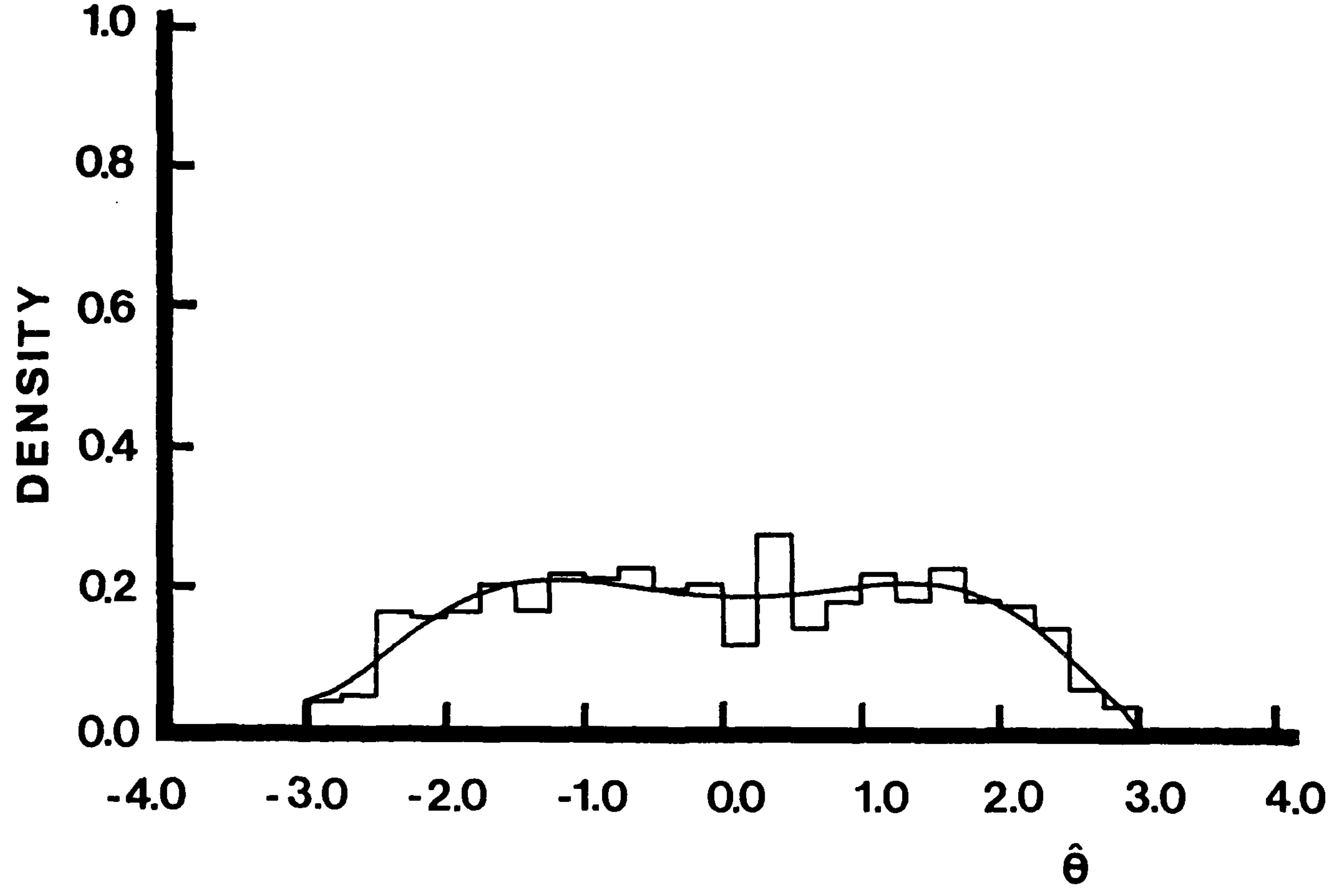

FIGURE 4-1 (Continued): Subtest 1, θ , Polynomial of Degree 6.

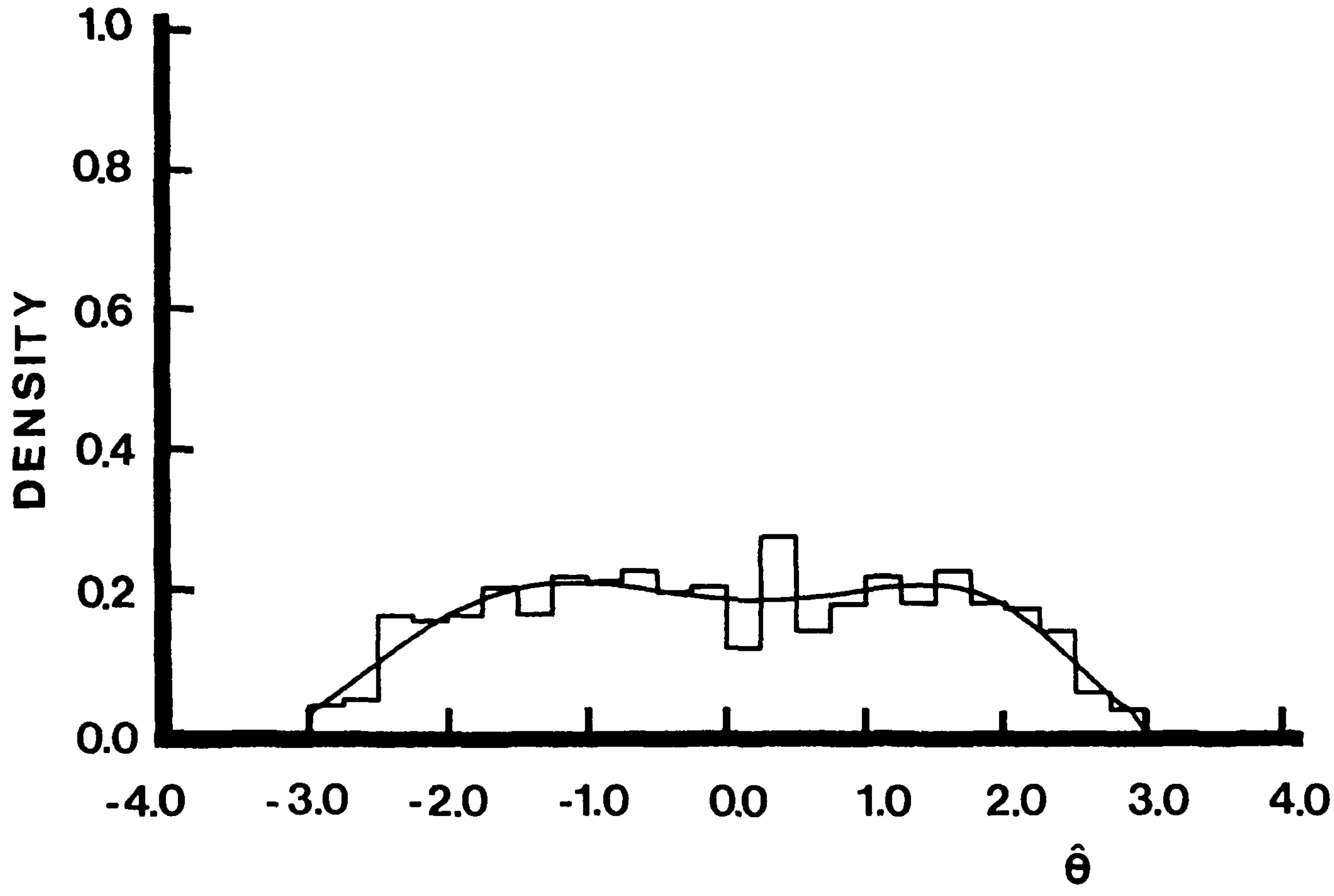

FIGURE 4-1 (Continued): Subtest 1, $\hat{\theta}$, Polynomial of Degree 7.

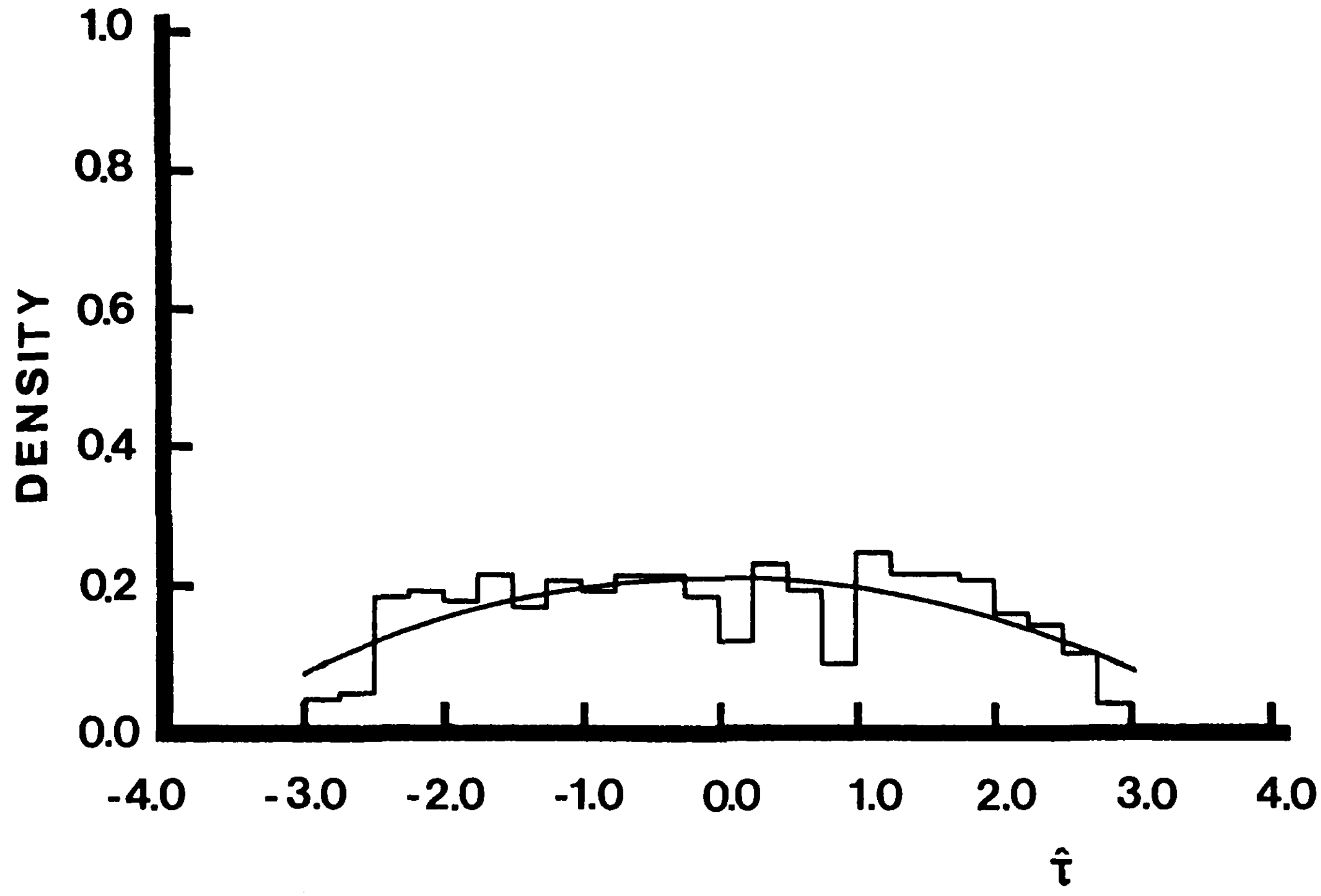

FIGURE 4-2

Relative Frequency Distribution of $\hat{\tau}$, Which Was Obtained for the Five Hundred Hypothetical Examinees on Subtest 1, with 0.25 as the Subinterval Width, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate the Density Function of $\hat{\tau}$.

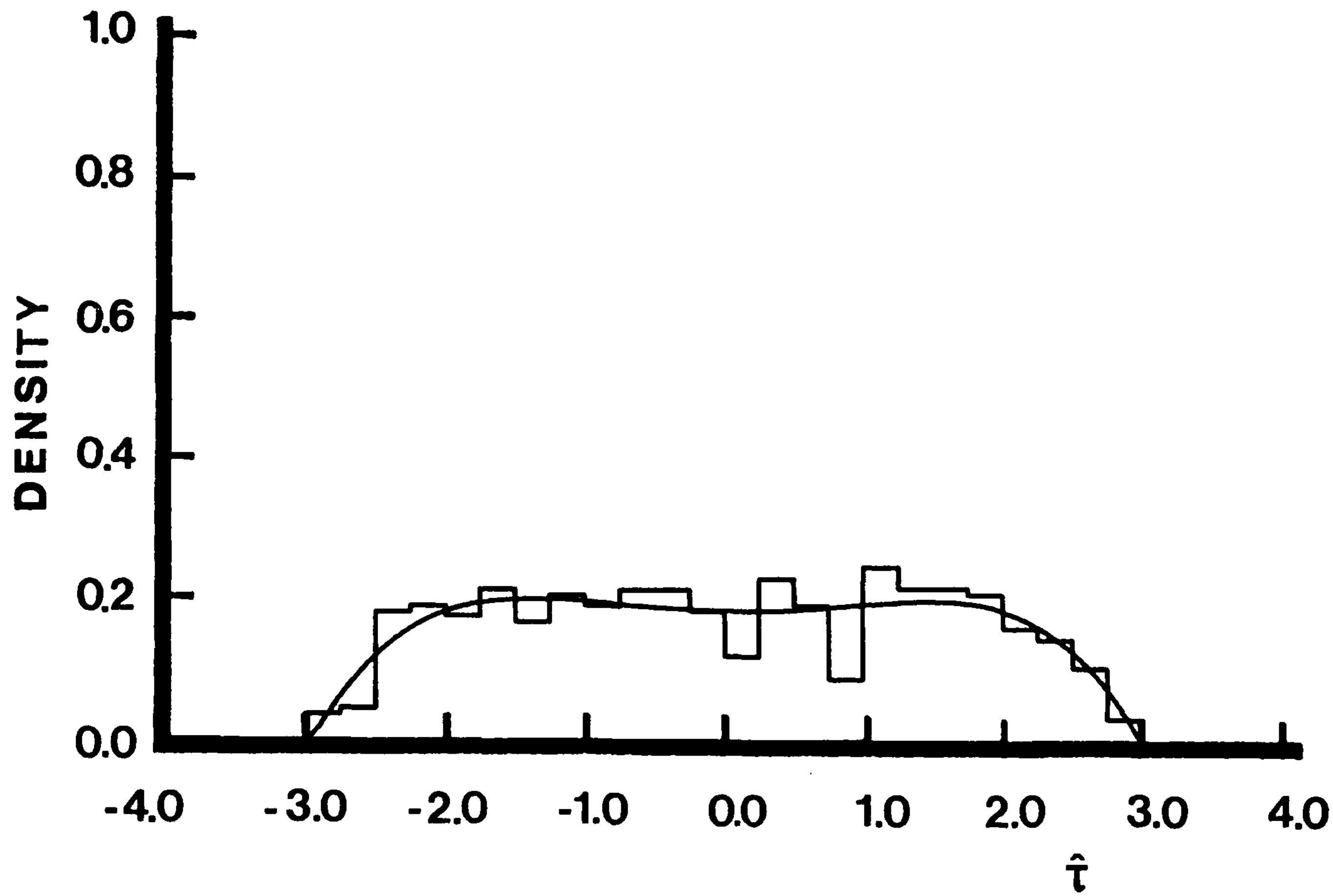

FIGURE 4-2 (Continued): Subtest 1, $\hat{\tau}$, Polynomial of Degree 4.

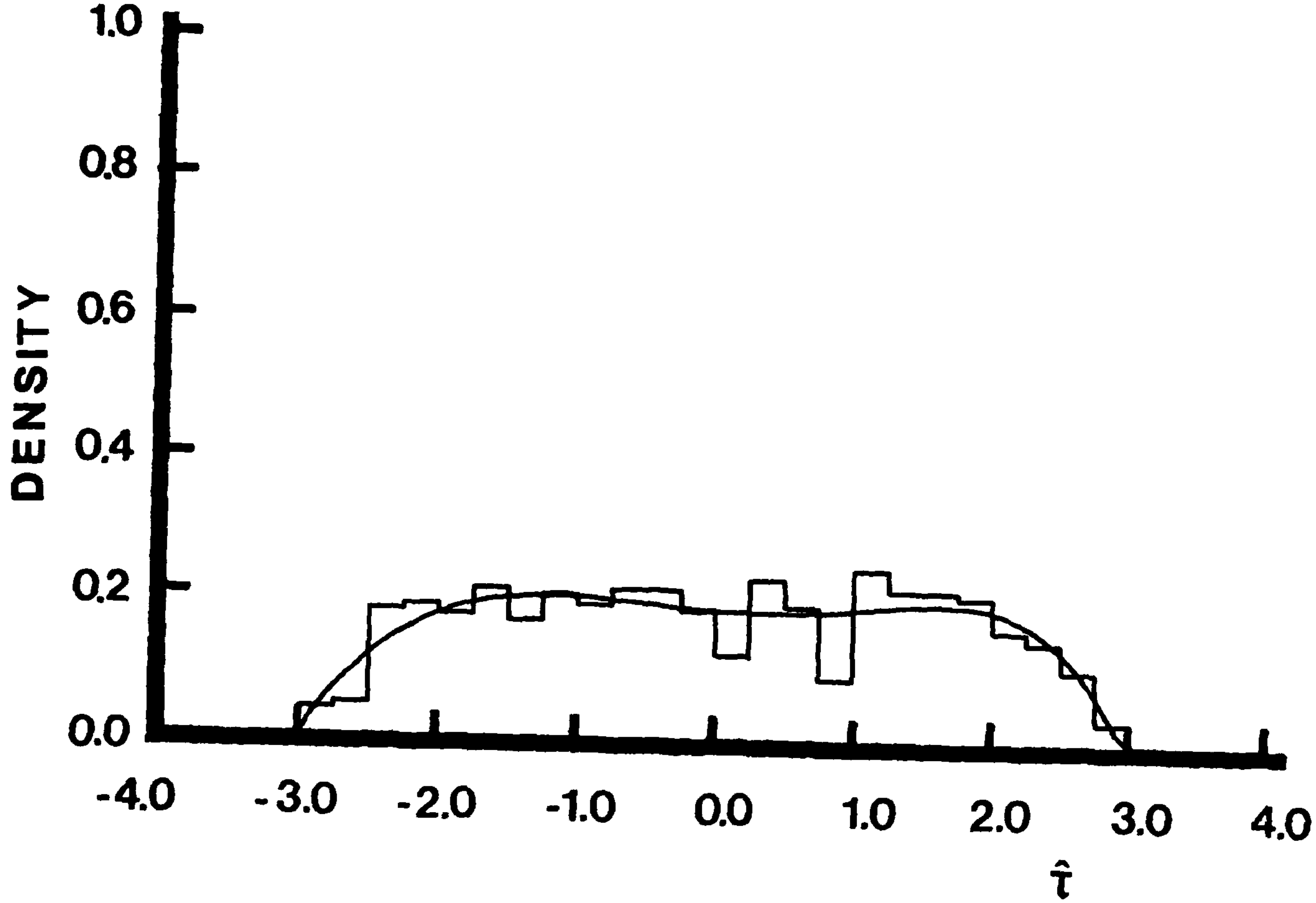

FIGURE 4-2 (Continued): Subtest 1, $\hat{\tau}$, Polynomial of Degree 5.

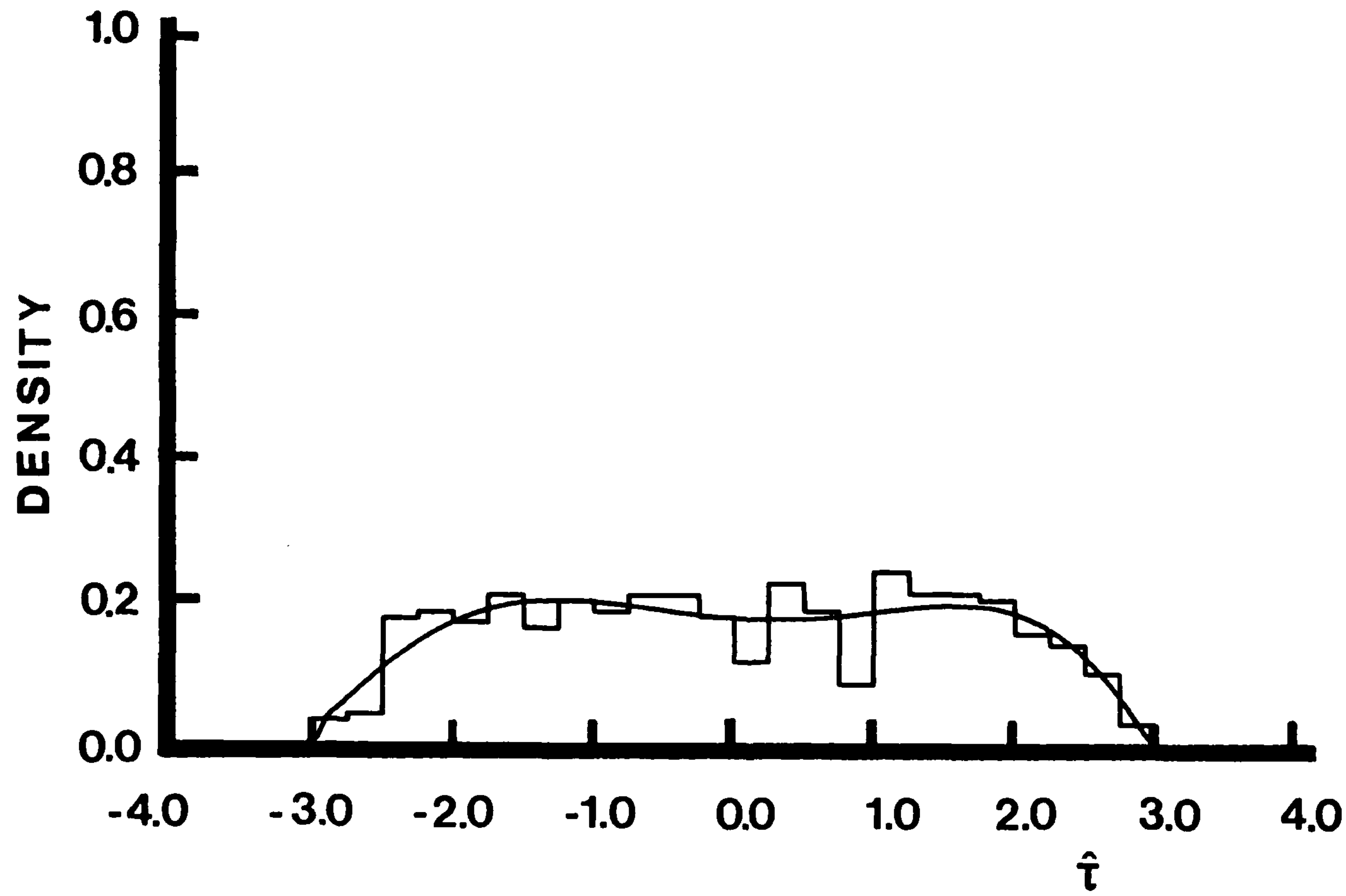

FIGURE 4-2 (Continued): Subtest 1, $\hat{\tau}$, Polynomial of Degree 6.

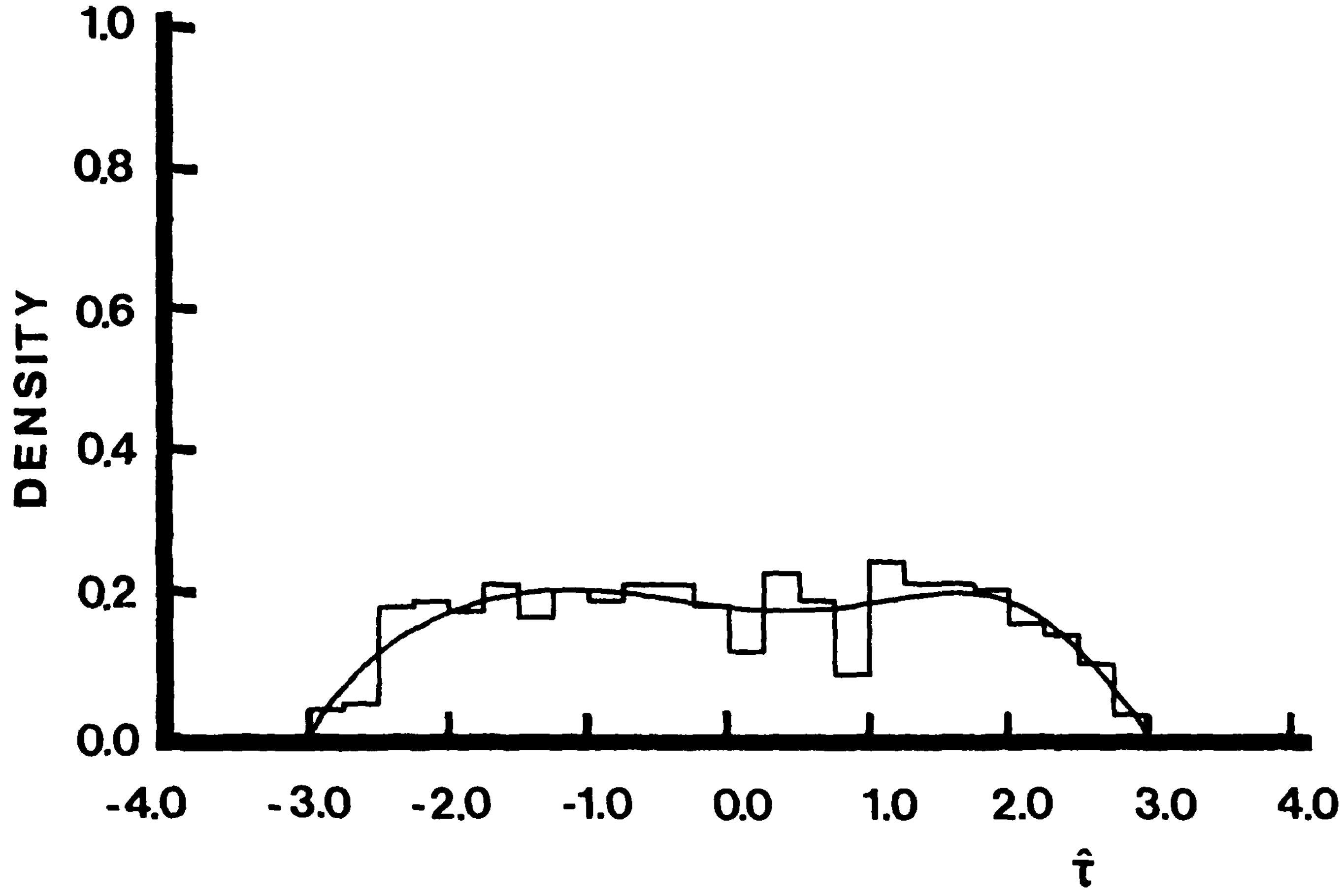

FIGURE 4-2 (Continued): Subtest 1, $\hat{\tau}$, Polynomial of Degree 7.

than that of $\hat{\theta}$, although they are similar in shape. To make the difference between the two frequency distributions more visible, five polynomials of degrees 3, 4, 5, 6 and 7 were obtained by the method of moments to approximate each of the density functions of $\hat{\theta}$ and $\hat{\tau}$, and were drawn by solid lines in the five graphs of each of Figures 4-1 and 4-2, along with the corresponding frequency distribution. We note that, except for the polynomial of degree 3 in each figure, the four approximated density functions are very similar to one another, and they are closer to a rectangle for $\hat{\tau}$ than those for $\hat{\theta}$. Since the method of moments was applied for a set of observations, instead of some empirical function, the 0-th through seventh moments about the origin were computed directly from the observations, and they turned out to be 1.00000, -0.00472, 2.19052, -0.04378, 9.17620, -0.52428, 48.47210 and -4.96487 for $\hat{\theta}$, and 1.00000, 0.00479, 2.12231, -0.02483, 8.51515, -0.35195, 42.31180 and -2.77758 for $\hat{\tau}$. The interval of $\hat{\theta}$ used for the method of moments is [-2.9843, 2.9904] and that of $\hat{\tau}$ is [-3.0479, 2.8681]. The coefficients of these ten polynomials are presented in Table 4-1.

Figures 4-3 and 4-4 present corresponding frequency distributions and the polynomials of degrees 3, 4, 5, 6 and 7 obtained through Subtest 2, respectively. This subtest also consists of twenty-five graded test items following the normal ogive model, but ten of the items are different from those which are used in

TABLE 4-1

Coefficients of the Two Sets of Polynomials of Degrees 3 Through 7, Which Were Obtained by the Method of Moments to Approximate the Density Functions of $\hat{\theta}$ and $\hat{\tau}$, Respectively. The Maximum Likelihood Estimation Is Based on Subtest 1.

		Coefficient for $\hat{\theta}$	Coefficient for $\hat{\tau}$
DGR. 3	0	0.22252	0.21204
	1	0.00090	-0.00092
	2	-0.01854	-0.01463
	3	-0.00023	0.00016
DGR. 4	0	0.19916	0.18470
	1	0.00074	-0.00198
	2	0.00765	0.01688
	3	-0.00019	0.00044
	4	-0.00342	-0.00424
DGR. 5	0	0.19918	0.18487
	1	-0.00609	-0.01220
	2	0.00761	0.01661
	3	0.00339	0.00594
	4	-0.00342	-0.00419
	5	-0.00036	-0.00057
DGR. 6	0	0.18920	0.18183
	1	-0.00623	-0.01244
	2	0.03108	0.02397
	3	0.00348	0.00611
	4	-0.01131	-0.00674
	5	-0.00037	-0.00059
	6	0.00065	0.00022
DGR. 7	0	0.18922	0.18198
	1	-0.01305	-0.02135
	2	0.03102	0.02351
	3	0.01036	0.01535
	4	-0.01128	-0.00654
	5	-0.00207	-0.00294
	6	0.00065	0.00020
	7	0.00012	0.00017

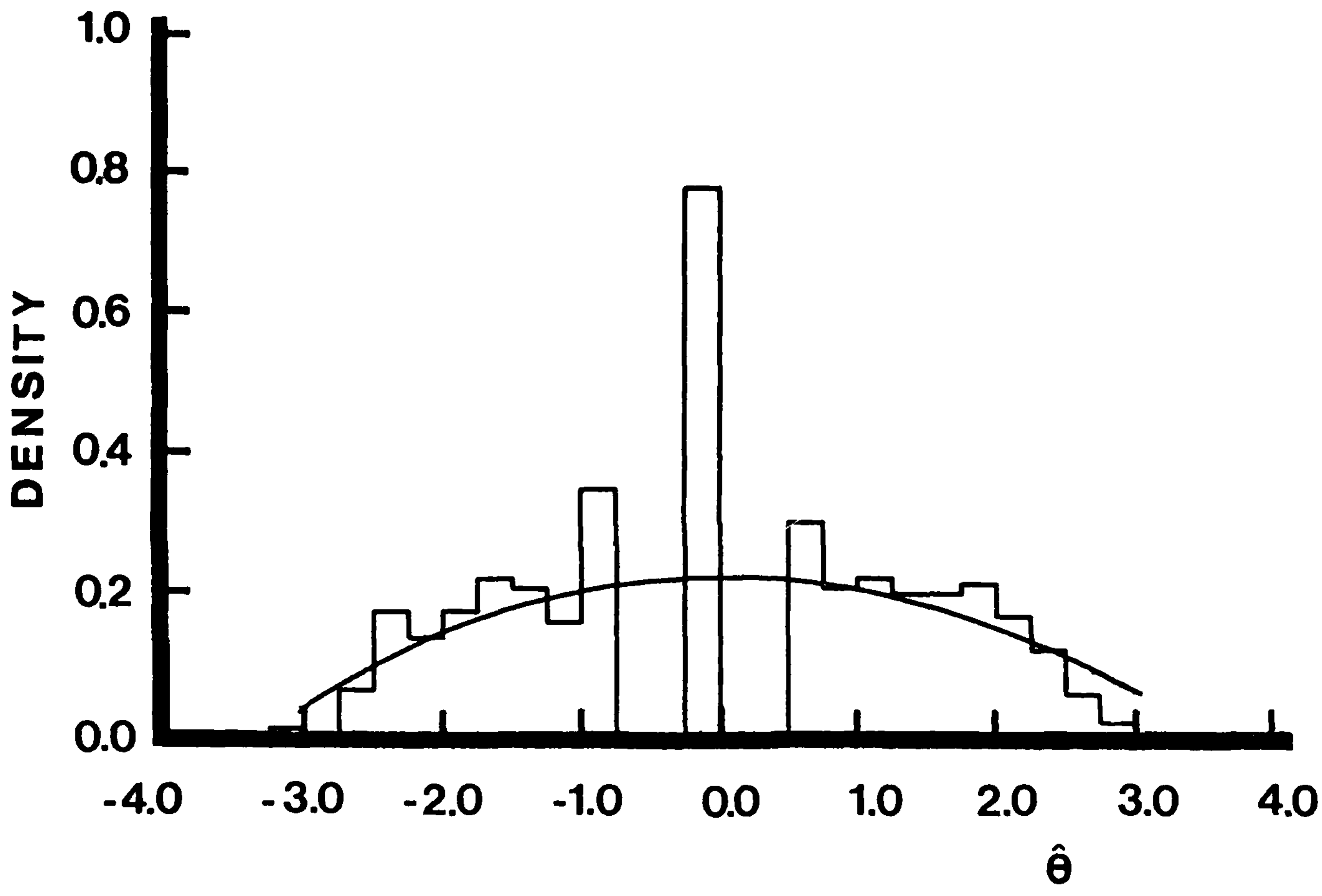

FIGURE 4-3

Relative Frequency Distribution of $\hat{\theta}$, Which Was Obtained for the Five Hundred Hypothetical Examinees on Subtest 2, with 0.25 as the Subinterval Width, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate the Density Function of θ .

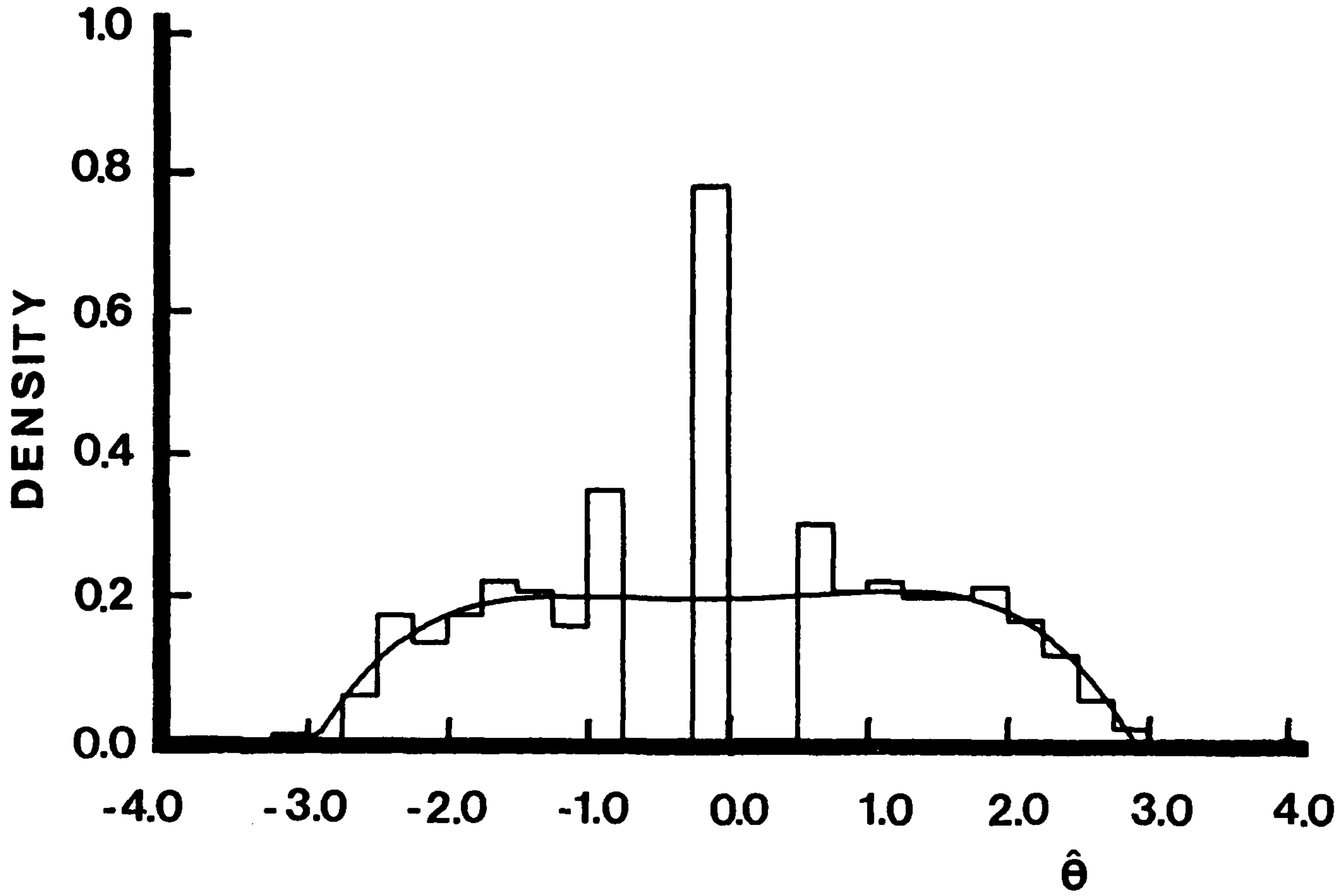

FIGURE 4-3 (Continued): Subtest 2, $\hat{\theta}$, Polynomial of Degree 4.

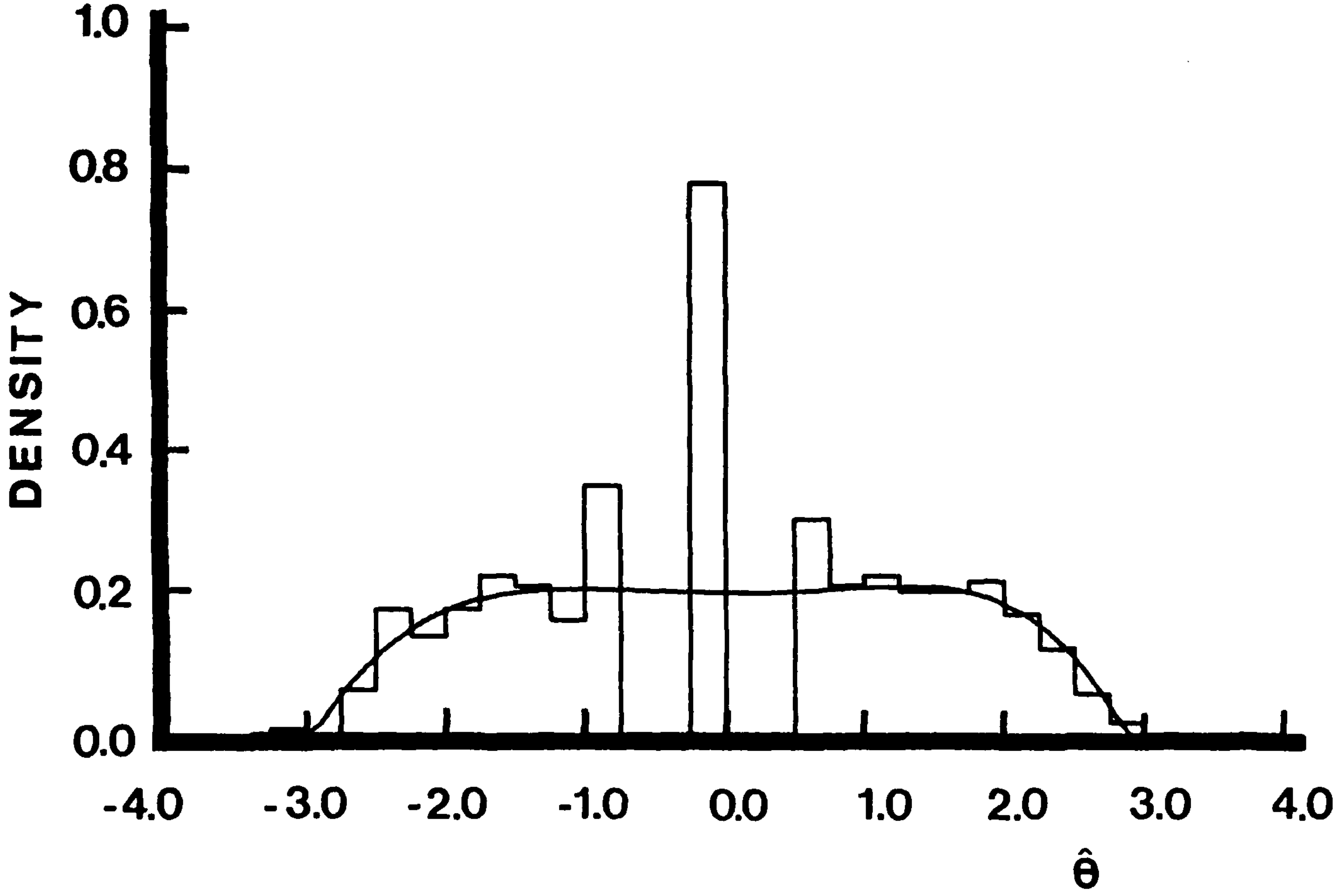

FIGURE 4-3 (Continued): Subtest 2, $\hat{\theta}$, Polynomial of Degree 5.

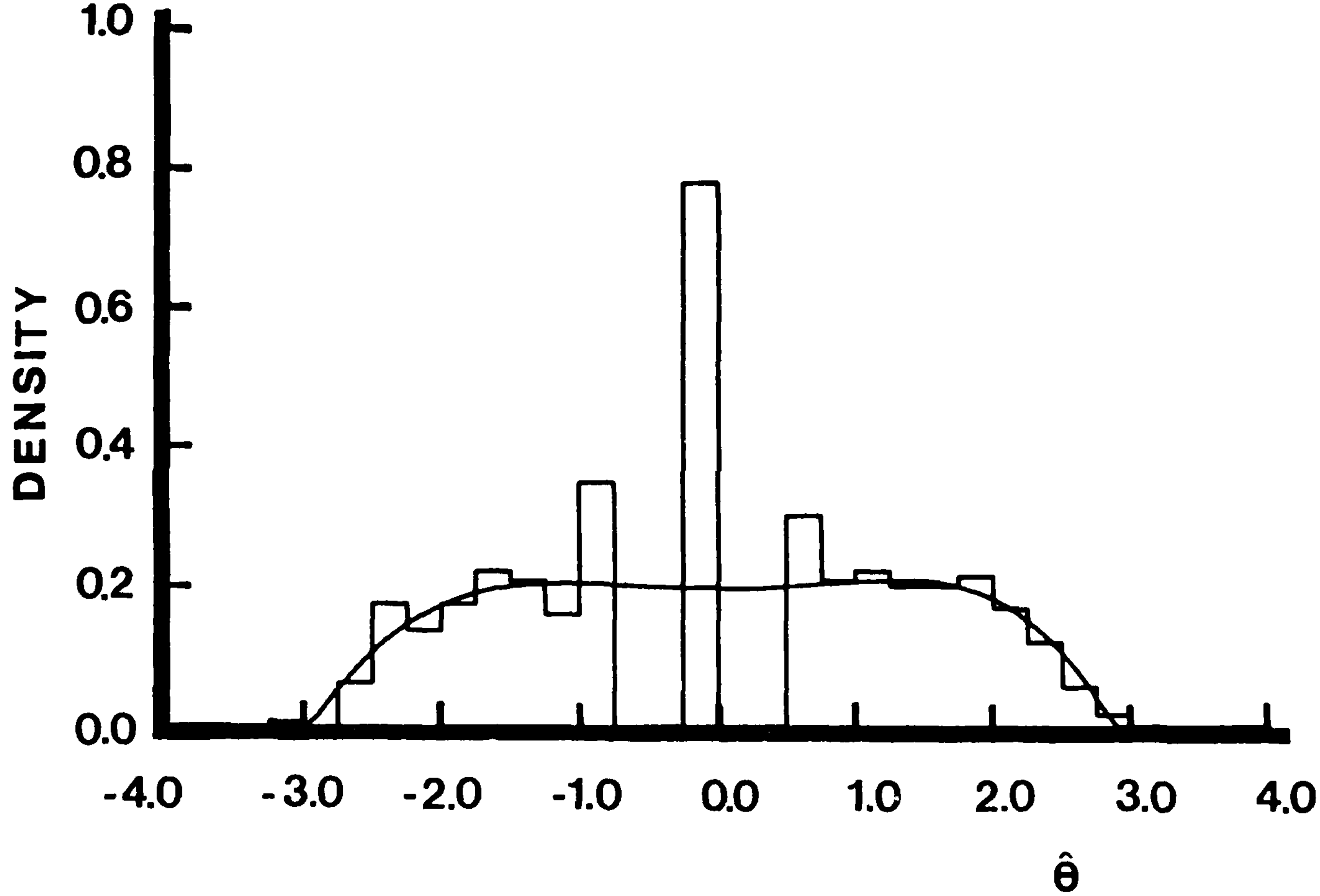

FIGURE 4-3 (Continued): Subtest 2, $\hat{\theta}$, Polynomial of Degree 6.

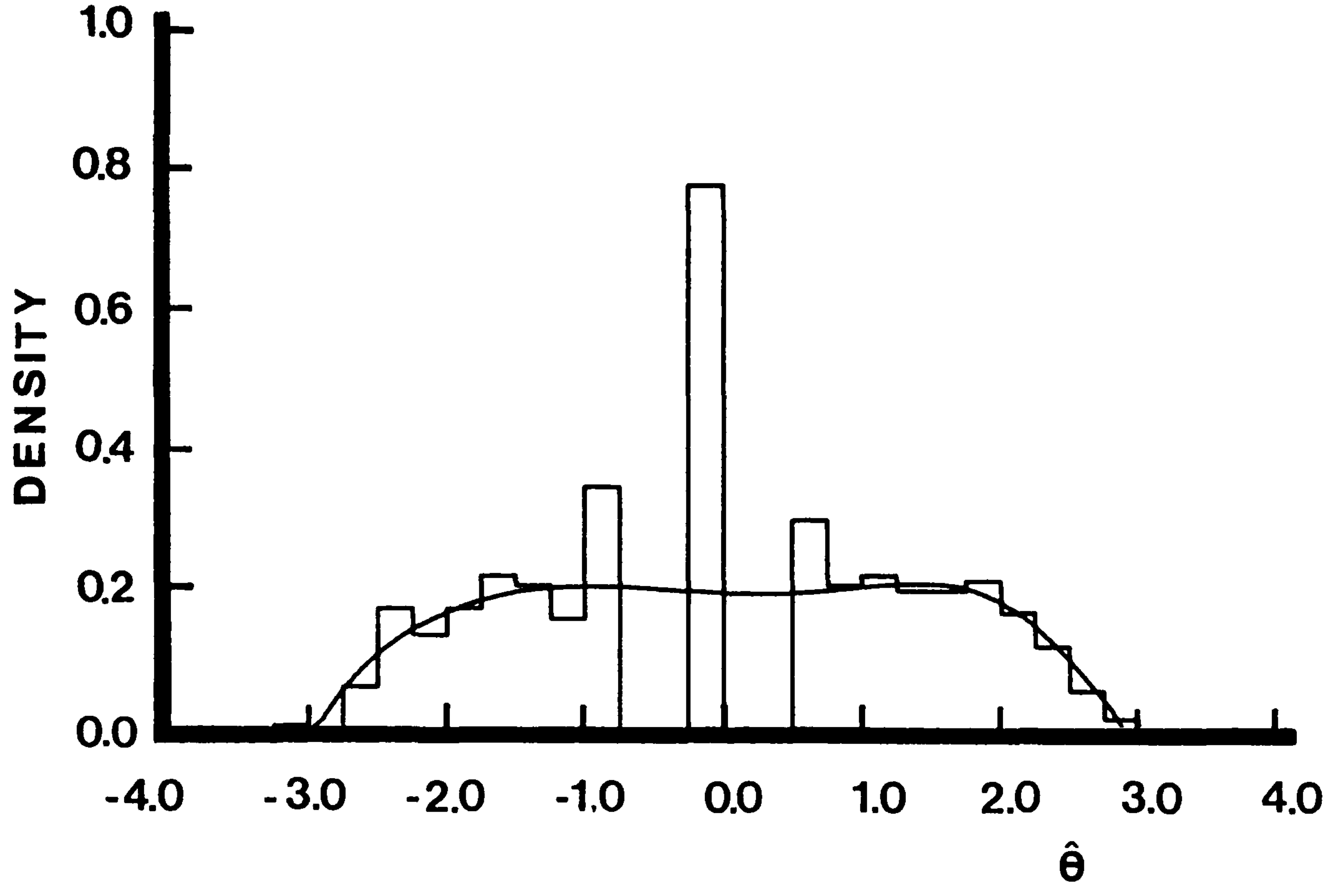

FIGURE 4-3 (Continued): Subtest 2, $\hat{\theta}$, Polynomial of Degree 7.

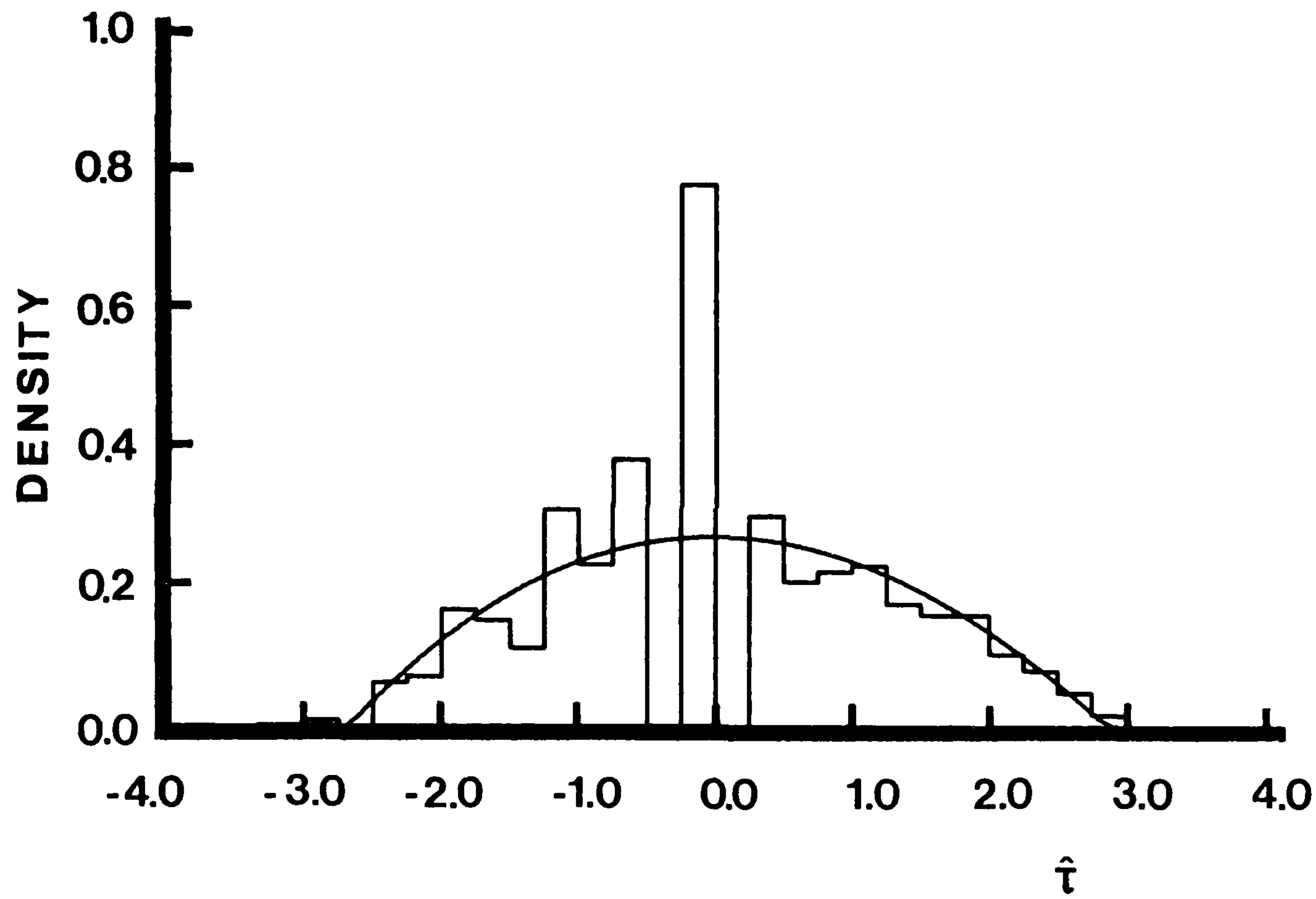

FIGURE 4-4

Relative Frequency Distribution of $\hat{\tau}$, Which Was Obtained for the Five Hundred Hypothetical Examinees on Subtest 2, with 0.25 as the Subinterval Width, Together with the Polynomial of Degree 3 Obtained by the Method of Moments to Approximate the Density Function of $\hat{\tau}$.

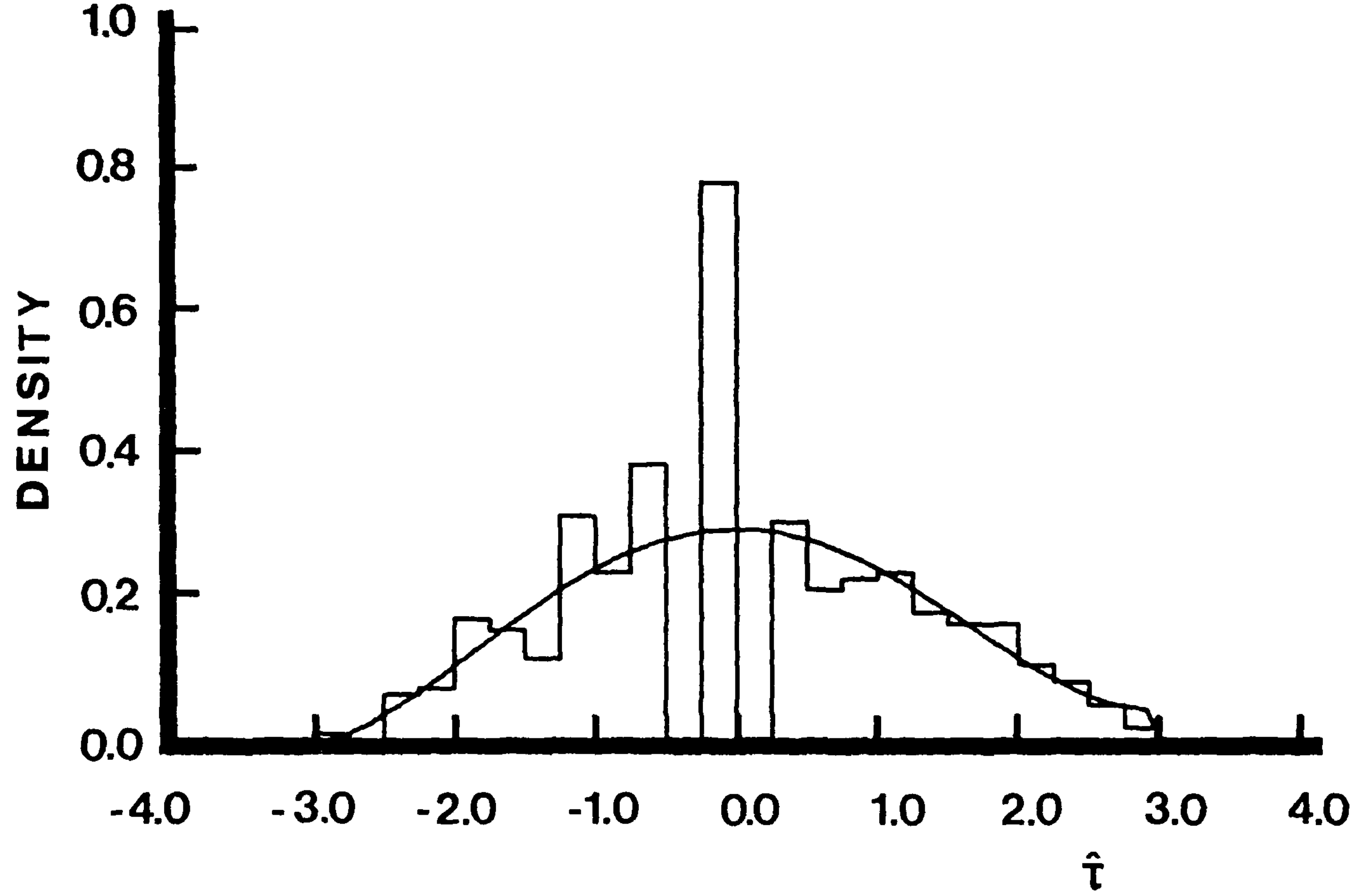

FIGURE 4-4 (Continued): Subtest 2, $\hat{\tau}$, Polynomial of Degree 4.

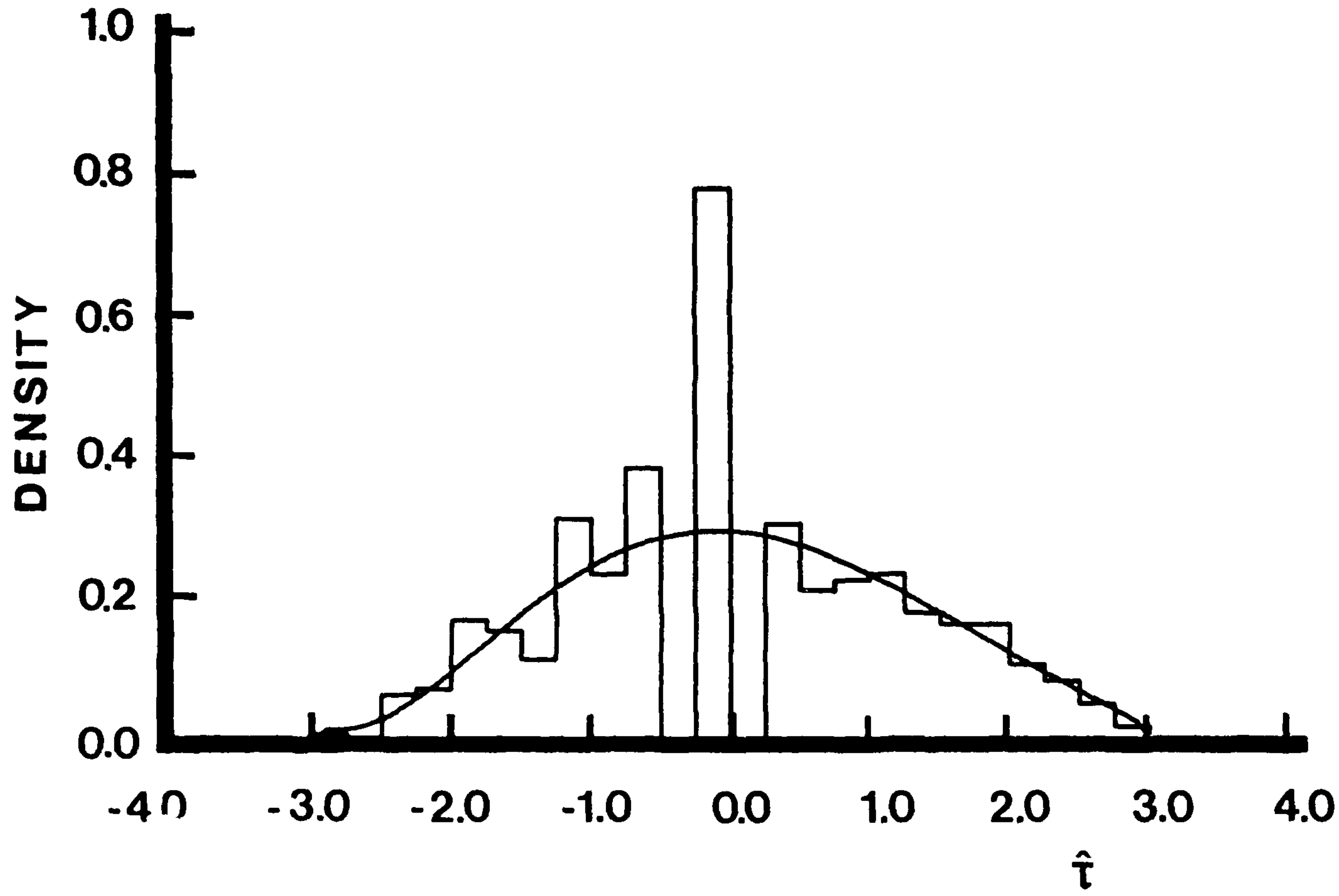

FIGURE 4-4 (Continued): Subtest 2, $\hat{\tau}$, Polynomial of Degree 5.

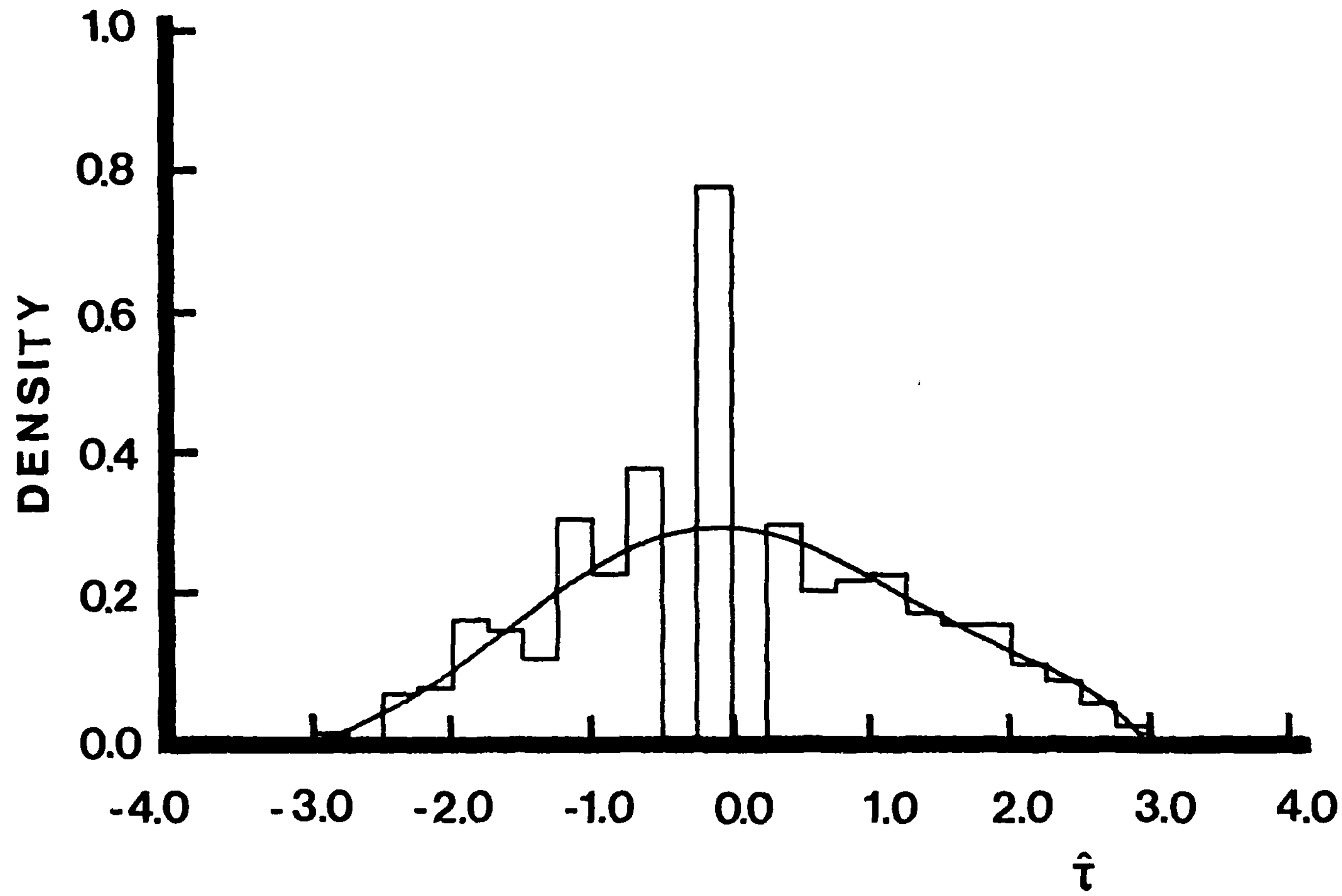

FIGURE 4-4 (Continued): Subtest 2, $\hat{\tau}$, Polynomial of Degree 6.

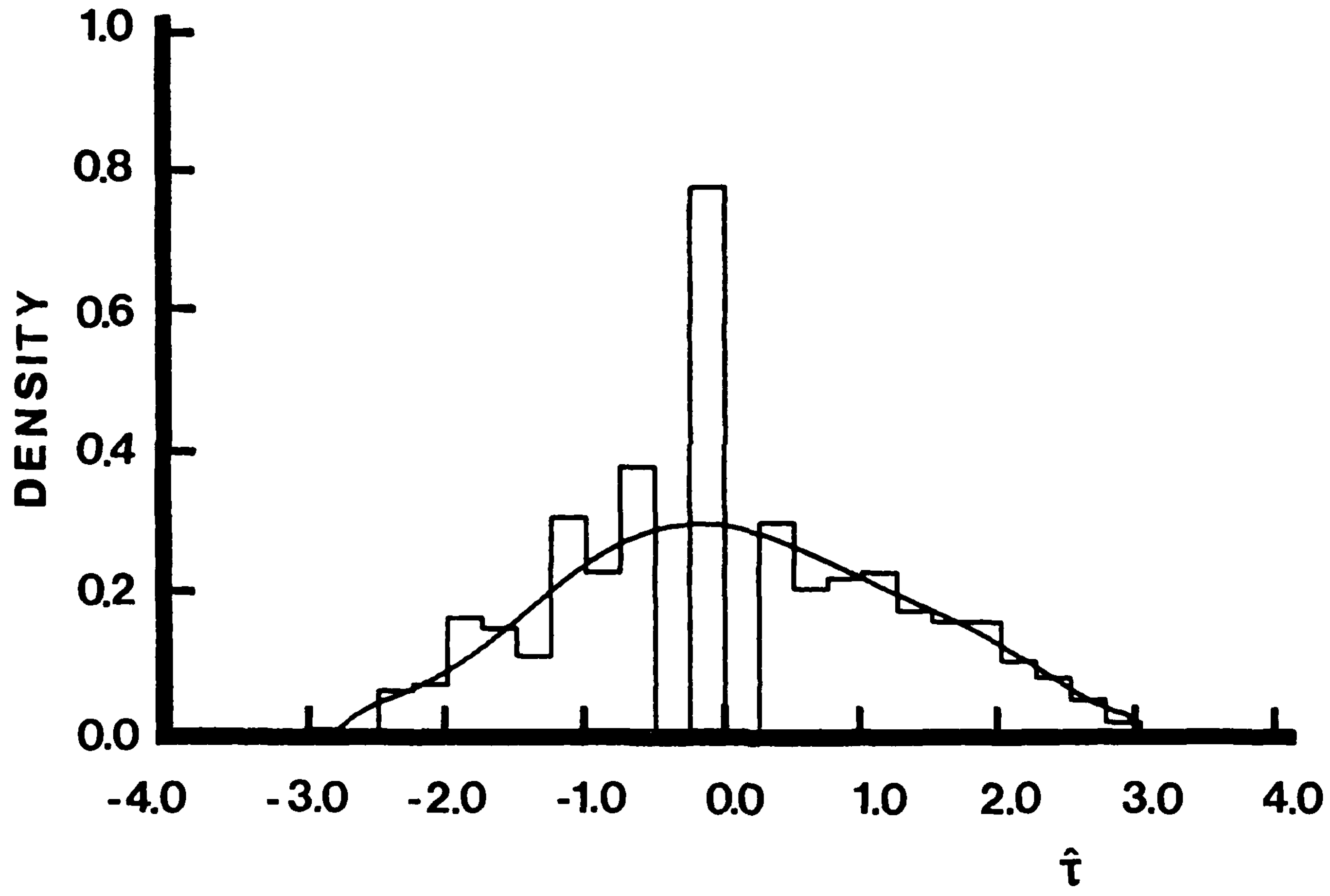

FIGURE 4-4 (Continued): Subtest 2, $\hat{\tau}$, Polynomial of Degree 7.

Subtest 1, as is shown in Tables 3-1 and 3-2. Just as in the case of Subtest 1, the transformation of $\hat{\theta}$ to $\hat{\tau}$ was made through (4.6) with $m = 7$, and the interval used for obtaining the coefficients α_k's in the method of moments is [-4.0, 4.0]. The coefficients α_k^*'s thus obtained are shown in Table 3-4. The amount of the constant test information for τ is different, however, and we used $C = 3.5$ instead of $C = 4.5$.

It is noted that the two frequency distributions of $\hat{\theta}$, which were obtained through Subtests 1 and 2, respectively, are substantially different from each other, and so is the case with those of $\hat{\tau}$. Although the latter is reasonable because of the difference in the two transformations of $\hat{\theta}$ to $\hat{\tau}$, the two frequency distributions of $\hat{\theta}$ should not be so different since they are both the estimates of the same θ for the same group of five hundred examinees. If we focus our attention on the polynomials approximating the density function of $\hat{\theta}$, however, we notice that the two sets of polynomials of degree 4 or greater are almost identical.

In each of Figures 4-3 and 4-4, the approximated polynomials are very similar, except for the one with degree 3, as was the case with those obtained through Subtest 1. These approximated density functions are steeper for $\hat{\tau}$ than for $\hat{\theta}$, and the difference is greater than in the case of Subtest 1. The 0-th through seventh moments about the origin for $\hat{\theta}$ are 1.00000, 0.00694, 2.31594, 0.07941, 9.95147, 0.41052, 52.81177 and 2.12395, and those for $\hat{\tau}$ are 1.00000,

0.06363, 1.48640, 0.41654, 5.19558, 2.54982, 24.35844 and 16.73911. The interval of $\hat{\theta}$ used in the method of moments is [-2.9290, 2.9625], and that of $\hat{\tau}$ is [-2.9315, 2.9160]. The coefficients of these polymomials are presented in Table 4-2.

TABLE 4-2

Coefficients of the Two Sets of Polynomials of Degrees 3 Through 7, Which Were Obtained by the Method of Moments to Approximate the Density Functions of $\hat{\theta}$ and $\hat{\tau}$, Respectively. The Maximum Likelihood Estimation Is Based on Subtest 2.

		Coefficient for $\hat{\theta}$	Coefficient for $\hat{\tau}$
0	DGR. 3	0.22600	0.27318
1		-0.00098	-0.00149
2		-0.01935	-0.03584
3		0.00057	0.00102
0	DGR. 4	0.19975	0.29301
1		0.00445	-0.00185
2		0.01073	-0.05903
3		-0.00089	0.00112
4		-0.00404	0.00317
0	DGR. 5	0.19932	0.29291
1		-0.00026	-0.01481
2		0.01141	-0.05887
3		0.00164	0.00819
4		-0.00415	0.00314
5		-0.00026	-0.00074
0	DGR. 6	0.19785	0.29859
1		0.00039	-0.01503
2		0.01496	-0.07282
3		0.00120	0.00834
4		-0.00538	0.00803
5		-0.00021	-0.00076
6		0.00010	-0.00042
0	DGR. 7	0.19707	0.29845
1		-0.00813	-0.03297
2		0.01737	-0.07238
3		0.01000	0.02724
4		-0.00639	0.00784
5		-0.00244	-0.00563
6		0.00020	-0.00040
7		0.00016	0.00035

V <u>Conditional Moments of the Maximum Likelihood Estimate</u> $\hat{\tau}$ <u>and the Three Methods of Approximating the Conditional Density</u> $\phi(\tau|\hat{\tau})$

Let λ be an estimator of τ, and η be the error of estimation. We assume that the conditional distribution of η, given τ, is normal, with 0 and σ as the two parameters, and λ is given by the simple sum of τ and η, such that

(5.1) $$\lambda = \tau + \eta \ .$$

We obtain for the first four conditional moments of τ about the origin, given λ,

(5.2) $$E(\tau|\lambda) = \lambda + \sigma^2 \frac{d}{d\lambda} \log g(\lambda) \ ,$$

(5.3) $$E(\tau^2|\lambda) = \lambda^2 + 2\lambda\sigma^2 \frac{d}{d\lambda} \log g(\lambda) + \sigma^4 [\frac{d^2}{d\lambda^2} \log g(\lambda) + \{\frac{d}{d\lambda} \log g(\lambda)\}^2] + \sigma^2 \ ,$$

(5.4) $$E(\tau^3|\lambda) = \sigma^6 [\frac{d^3}{d\lambda^3} \log g(\lambda)] \ ,$$

and

(5.5) $$E(\tau^4|\lambda) = \sigma^4 [3 + 6\sigma^2 \{\frac{d^2}{d\lambda^2} \log g(\lambda)\} + 3\sigma^4 \{\frac{d^2}{d\lambda^2} \log g(\lambda)\}^2 + \sigma^4 \{\frac{d^4}{d\lambda^4} \log g(\lambda)\}] \ ,$$

where $g(\lambda)$ is the marginal density function of λ.

By virtue of the fact that $I^*(\tau) = C^2$ and that the asymptotic conditional distribution of the maximum likelihood estimate $\hat{\tau}$, given τ, is the normal distribution with τ and $[I^*(\tau)]^{-1/2}$ as the

parameters (Samejima, 1975), we can write for the first four conditional moments of τ about the origin, given $\hat{\tau}$,

(5.6) $$E(\tau|\hat{\tau}) = \hat{\tau} + C^{-2}\frac{d}{d\hat{\tau}} \log g(\hat{\tau}) ,$$

(5.7) $$E(\tau^2|\hat{\tau}) = \hat{\tau}^2 + 2\hat{\tau}C^{-2} \frac{d}{d\tau} \log g(\hat{\tau}) + C^{-4}[\frac{d^2}{d\hat{\tau}^2} \log g(\hat{\tau}) + \{\frac{d}{d\hat{\tau}} \log g(\hat{\tau})\}^2] + C^{-2} ,$$

(5.8) $$E(\tau^3|\hat{\tau}) = C^{-6}[\frac{d^3}{d\hat{\tau}^3} \log g(\hat{\tau})] ,$$

(5.9) $$E(\tau^4|\hat{\tau}) = C^{-4}[3 + 6C^{-2}\{\frac{d^2}{d\hat{\tau}^2} \log g(\hat{\tau})\} + 3C^{-4}\{\frac{d^2}{d\hat{\tau}^2} \log g(\hat{\tau})\}^2 + C^{-4}\{\frac{d^4}{d\hat{\tau}^4} \log g(\hat{\tau})\}] ,$$

where $g(\hat{\tau})$ is the marginal density function of $\hat{\tau}$.

The formulas (5.6) through (5.9) imply that, since the set of N maximum likelihood estimates, $\hat{\tau}$, is available as our basic data, these conditional moments can solely be estimated from $g(\hat{\tau})$, provided that we can approximate this marginal density function by fitting an appropriate four-time differentiable function to the set of N $\hat{\tau}$'s . This has been done in the previous studies using θ instead of τ , by adopting a polynomial of degree 3 or 4, which was obtained by the method of moments.

After these conditional moments have been obtained, which are functions of $\hat{\tau}$, we can fit some appropriate function for the conditional densitv function of τ , given $\hat{\tau}$. In the Normal Approach Method, only the first two conditional moments are used,

and the normal density function is fitted for the conditional distribution with $E(\tau|\hat{\tau})$ and $[E(\tau^2|\hat{\tau}) - \{E(\tau|\hat{\tau})\}^2]^{1/2}$ as the parameters. For simplicity, let μ_1' be the first conditional moment of τ about the origin, and μ_2 be the second conditional moment of τ about the mean, given $\hat{\tau}$, respectively. Thus the approximated conditional density function, $\hat{\phi}(\tau|\hat{\tau})$, in the Normal Approach Method is given by

$$(5.10) \qquad \hat{\phi}(\tau|\hat{\tau}) = (2\pi\mu_2)^{-1/2} \exp[-(\tau-\mu_1')^2/(2\mu_2)] \quad .$$

In the Pearson-System Method, all of the above four conditional moments are used. For simplicity, let μ_3 and μ_4 denote the third and fourth conditional moments of τ about the mean, given $\hat{\tau}$, adding to the symbols, μ_1' and μ_2. Pearson's criterion κ (Elderton and Johnson, 1969; Johnson and Kotz, 1970) is defined by

$$(5.11) \qquad \kappa = \beta_1(\beta_2+3)^2[4(2\beta_2-3\beta_1-6)(4\beta_2-3\beta_1)]^{-1} \; ,$$

where β_1 and β_2 are given by

$$(5.12) \qquad \beta_1 = \mu_3^2\mu_2^{-3}$$

and

$$(5.13) \qquad \beta_2 = \mu_4\mu_2^{-2} \; .$$

Depending upon the value of κ, one of the Pearson type distributions is assigned as the approximation to the conditional distribution of τ, given $\hat{\tau}$. For different values of $\hat{\tau}$, therefore, possibly different

types of Pearson distributions are assigned, and we have varieties of different types of density functions for $\hat{\phi}(\tau|\hat{\tau})$. If, for instance, $\kappa < 0$, then the distribution assigned is the Beta distribution, whose density function is given by the formula

$$\text{(5.14)} \qquad \hat{\phi}(\tau|\hat{\tau}) = [B(p_{\hat{\tau}},q_{\hat{\tau}})]^{-1} (\tau-a_{\hat{\tau}})^{p_{\hat{\tau}}-1}(b_{\hat{\tau}}-\tau)^{q_{\hat{\tau}}-1}(b_{\hat{\tau}}-a_{\hat{\tau}})^{-(p_{\hat{\tau}}+q_{\hat{\tau}}-1)} ,$$

in which the four parameters, $p_{\hat{\tau}}$, $q_{\hat{\tau}}$, $a_{\hat{\tau}}$, and $b_{\hat{\tau}}$, are estimated from the four conditional moments, such that

$$\text{(5.15)} \qquad p_{\hat{\tau}}, q_{\hat{\tau}} = (r/2)[1 \pm (r+2)\{\beta_1[\beta_1(r+2)^2 + 16(r+1)]^{-1}\}^{1/2}] ,$$

$$\text{(5.16)} \qquad \hat{b}_{\hat{\tau}} - \hat{a}_{\hat{\tau}} = \mu_2^{1/2}[\beta_1(r+2)^2 + 16(r+1)]^{1/2}/2 ,$$

$$\text{(5.17)} \qquad \hat{a}_{\hat{\tau}} = \mu_1' - \hat{p}_{\hat{\tau}}(\hat{b}_{\hat{\tau}}-\hat{a}_{\hat{\tau}})/r ,$$

and

$$\text{(5.18)} \qquad \hat{b}_{\hat{\tau}} = \mu_1' + q_{\hat{\tau}}(\hat{b}_{\hat{\tau}}-\hat{a}_{\hat{\tau}})/r ,$$

where r is defined as

$$\text{(5.19)} \qquad r = 6(\beta_2-\beta_1-1)(6+3\beta_1-2\beta_2)^{-1} .$$

If $\kappa = 0$, which results from $\beta_1 = 0$ and $\beta_2 < 3$, the distribution is a special case of Beta distribution in which the density function is symmetric, and two parameters, $p_{\hat{\tau}}$ and $q_{\hat{\tau}}$, are equal, such that

$$\text{(5.20)} \qquad \hat{p}_{\hat{\tau}} = \hat{q}_{\hat{\tau}} = r/2 .$$

If $\kappa = 0$, which is resultant from $\beta_1 = 0$ and $\beta_2 = 3$, then the normal distribution is assigned, whose density function is given by (5.10). If $\kappa > 1$, then the distribution is of Pearson's Type VI, and, if $0 < \kappa < 1$, then the distribution is of Pearson's Type IV, and so forth.

The advantage of Pearson-System Method over the other two methods is that it makes full use of the four estimated conditional moments of τ , given $\hat{\tau}$, without restricting the conditional distributions to a single type. It has its disadvantage, however, since in some cases the estimation of the higher conditional moments is fairly inaccurate for some range of $\hat{\tau}$, and also the estimation of the parameters of some Pearson type distributions is difficult.

In the <u>Two-Parameter Beta Method</u>, the Beta distribution is adopted for the conditional distribution of τ , given $\hat{\tau}$, whose density function is given by (5.14). Two parameters, $a_{\hat{\tau}}$ and $b_{\hat{\tau}}$, are preassigned for each $\hat{\tau}$ in some appropriate method, and the other two parameters, $p_{\hat{\tau}}$ and $q_{\hat{\tau}}$, are estimated by

(5.21) $$\hat{p}_{\hat{\tau}} = M_1^2(1-M_1)M_2^{-1} - M_1$$

and

(5.22) $$\hat{q}_{\hat{\tau}} = M_1(1-M_1)^2M_2^{-1} - (1-M_1) \quad ,$$

where

(5.23) $$M_1 = (\mu_1'-a_{\hat{\tau}})(b_{\hat{\tau}}-a_{\hat{\tau}})^{-1}$$

and

$$(5.24) \qquad M_2 = \mu_2(b_{\hat{\tau}}-a_{\hat{\tau}})^{-2}$$

This method has an advantage over the Normal Approach Method in the sense that, unlike the normal density function, the Beta density function provides us with varieties of different curves depending upon the values of the parameters. Its disadvantage is, however, that we have an additional work of finding an appropriate finite interval, $[a_{\hat{\tau}}, b_{\hat{\tau}}]$.

VI Histogram Ratio and Curve Fitting Approaches

The two approaches discussed here, as well as Conditional P.D.F. Approach, make full use of the approximated density function, $\hat{g}(\hat{\tau})$, which is obtained on the entire set of N $\hat{\tau}$'s . The conditional moments of τ , given $\hat{\tau}$, are obtained by (5.6) through (5.9), using this approximated density function for $g(\hat{\tau})$.

We calibrate a certain number of τ for each of the N $\hat{\tau}$'s, through the Monte Carlo method, in accordance with the approximated conditional density function of τ , given $\hat{\tau}$. This approximated density function, $\hat{\phi}(\tau|\hat{\tau})$, can be a normal density function, a Beta density function, or one of the Pearson System density functions, depending upon which of the three methods, i.e., Normal Approach Method, Two-Parameter Beta Method and Pearson-System Method, we choose. Let $\tilde{\tau}$ denote these calibrated τ's , and ν be the number of $\tilde{\tau}$'s calibrated for each $\hat{\tau}_i$ of examinee i . Thus we obtain $(\nu x N)$ $\tilde{\tau}$'s in total. We classify these $\tilde{\tau}$'s into (m_h+1) item score groups, where h is a new test item whose operating characteristics are to be estimated, depending upon the item score x_h $(=0,1,\ldots,m_h)$ the specific examinee obtained for item h . Then each $\tilde{\tau}$ is transformed to $\tilde{\theta}$, through

$$(6.1) \qquad \theta = \tau^{-1}[\tau(\theta)] \ .$$

When $\tau(\ )$ is given by the polynomial given by (3.5), for example, this process can easily be performed by Newton-Raphson Method.

In the Histogram Ratio Approach, these $(\nu \times N)$ $\tilde{\theta}$'s are categorized into intervals of small, equal widths. The ratio of the frequency of $\tilde{\theta}$'s, which belong to examinees whose item score to item h is x_h, to the total frequency, in each subinterval of θ, provides us with the estimated operating characteristic, $\hat{P}_{x_h}(\theta)$. Let $H_{x_h}(\tilde{\theta}\varepsilon s)$ denote the frequency of $\tilde{\theta}$'s, which belong to the item score group x_h, for the subinterval s, whose midpoint is θ_s. Then we can write

$$(6.2) \qquad \hat{P}_{x_h}(\theta_s) = H_{x_h}(\tilde{\theta}\varepsilon s)\left[\sum_{j=0}^{m_h} H_j(\tilde{\theta}\varepsilon s)\right]^{-1}, \qquad x_h = 0,1,\ldots,m_h .$$

In order to obtain a smooth curve for this estimated operating characteristic, it is advisable to use a fairly large number for ν, and a small width for the subinterval s of θ.

In the Curve Fitting Approach, a polynomial of a certain degree is fitted by the method of moments, to the subset of $\tilde{\theta}$'s for each item score group x_h. Then the ratio of the resultant polynomial to the sum of (m_h+1) such polynomials is taken, and this ratio provides us with the estimated operating characteristic of the item response x_h. Let $\eta_{x_h}(\theta)$ be such a polynomial for the item score group x_h. We obtain for the estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, such that

$$(6.3) \qquad \hat{P}_{x_h}(\theta) = \eta_{x_h}(\theta)\left[\sum_{j=0}^{m_h} \eta_j(\theta)\right]^{-1}, \qquad x_h = 0,1,\ldots,m_h .$$

VII Conditional P.D.F. Approach

In this approach, we specify the exact function of the approximated conditional density, $\hat{\phi}(\tau|\hat{\tau})$, using the parameters estimated from the approximated density function $\hat{g}(\hat{\tau})$, (cf. Chapter 5). Again, this approximation to the conditional density function, $\hat{\phi}(\tau|\hat{\tau})$, can be a normal density function, a Beta density function, or one of the Pearson System density functions, depending upon which one of the Normal Approach Method, the Two-Parameter Beta Method, and the Pearson-System Method we choose.

In the Simple Sum Procedure, these specified, approximated conditional density functions are categorized into the (m_h+1) item score groups for a new item h , whose operating characteristics are to be estimated, depending upon the item score x_h $(=0,1,2,\ldots,m_h)$ that each examinee has obtained. By virtue of (2.10), the transformation of τ to θ is made through (6.1), and the estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, is given by

$$\hat{P}_{x_h}(\theta) = \sum_{i\varepsilon x_h} \hat{\phi}(\tau|\hat{\tau}_i)[\sum_{i=1}^{N} \hat{\phi}(\tau|\hat{\tau}_i)]^{-1} , \qquad x_h = 0,1,\ldots,m_h , \tag{7.1}$$

where i denotes an individual examinee and $\hat{\tau}_i$ is the maximum likelihood estimate of τ for the individual i .

In the Weighted Sum Procedure, the estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, of the item response x_h can be written as

$$\hat{P}_{x_h} = \sum_{i\varepsilon x_h} w(\hat{\tau}_i) \hat{\phi}(\tau|\hat{\tau}_i)[\sum_{i=1}^{N} w(\hat{\tau}_i)\hat{\phi}(\tau|\hat{\tau}_i)]^{-1} , \tag{7.2}$$

$$x_h = 0,1,\ldots,m_h ,$$

where $w(\hat{\tau}_i)$ is an appropriate weight assigned to the maximum likelihood estimate $\hat{\tau}$ for the individual examinee i. Simple Sum Procedure can be considered, therefore, as a special case of the Weighted Sum Procedure, in which $w(\hat{\tau}_i) = 1$ for all the individual examinees. Another example of such a weight, $w(\hat{\tau}_i)$, is the area under the approximated density function, $\hat{g}(\hat{\tau})$, for the interval of $\hat{\tau}$ which starts from the midway between $\hat{\tau}_i$ and the lower adjacent $\hat{\tau}_i$ and ends with the midway between $\hat{\tau}_i$ and the upper adjacent $\hat{\tau}_i$. The transformation of τ to θ in (7.2) can be made through (6.1), as in the Simple Sum Procedure.

We have a somewhat different rationale behind the Proportioned Sum Procedure. Let $p(i \varepsilon x_h)$ be the probability with which examinee i belongs to the item score group x_h. We can write for the estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, of the item response x_h to a new item h

$$(7.3) \qquad \hat{P}_{x_h}(\theta) = \sum_{i=1}^{N} \hat{p}(i \varepsilon x_h)\, \hat{\phi}(\tau|\hat{\tau}_i)\, [\sum_{i=1}^{N} \hat{\phi}(\tau|\hat{\tau}_i)]^{-1} ,$$

$$x_h = 0,1,\ldots,m_h$$

where $\hat{p}(i \varepsilon x_h)$ is the estimate of the probability $p(i \varepsilon x_h)$, which satisfies

$$(7.4) \qquad \sum_{x_h=0}^{m_h} \hat{p}(i \varepsilon x_h) = \sum_{x_h=0}^{m_h} p(i \varepsilon x_h) = 1 .$$

One example of this proportional weight, $\hat{p}(i \varepsilon x_h)$, is the proportion of examinees who belong to the item score group x_h within a specified

interval of $\hat{\tau}$ for which $\hat{\tau}_i$ is the midpoint. The transformation of τ to θ in (7.3) is, again, made through (6.1).

VIII Bivariate P.D.F. Approach

In contrast to the other three approaches, Bivariate P.D.F. Approach makes use of the estimated bivariate density function, rather than the estimated conditional density function, $\hat{\phi}(\tau|\hat{\tau})$. Let $\xi(\hat{\tau},\tau)$ denote the bivariate density function of $\hat{\tau}$ and τ . We can write

$$(8.1) \qquad \xi(\hat{\tau},\tau) = \phi(\tau|\hat{\tau})\, g(\hat{\tau}) \text{ .}$$

We classify the set of N $\hat{\tau}_i$'s into (m_h+1) item score categories, depending upon the item score x_h $(=0,1,\ldots,m_h)$ the examinee i obtained for a new test item h , for which the operating characteristics are to be estimated.

The method of moments is applied for each of these (m_h+1) subsets of $\hat{\tau}$, and the density function, $g_{x_h}(\hat{\tau})$, is estimated for each subgroup. The conditional moments of τ , given $\hat{\tau}$, are also obtained for separate subgroups, using the formulas (5.6) through (5.9). Based on these estimated conditional moments, the parameters of a specific density function, which is adopted for $\phi(\tau|\hat{\tau})$, are obtained for each subgroup x_h . The choice of $\hat{\phi}(\tau|\hat{\tau})$ depends upon which of the three methods, i.e., Normal Approach Method, Two-Parameter Beta Method and Pearson-System Method, is taken. The bivariate density function of $\hat{\tau}$ and τ is obtained from (8.1) for each of the (m_h+1) subgroups. Let $\hat{\xi}_{x_h}(\hat{\tau},\tau)$ denote the estimated bivariate density function of $\hat{\tau}$ and τ for the subgroup

x_h . The estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, is given by

$$(8.2) \qquad \hat{P}_{x_h}(\theta) = \int_{-\infty}^{\infty} \hat{\xi}_{x_h}(\hat{\tau},\tau)d\hat{\tau} \, [\sum_{j=0}^{m_h} \int_{-\infty}^{\infty} \hat{\xi}_j(\hat{\tau},\tau)d\hat{\tau}]^{-1} \quad , \quad x_h = 0,1,\ldots,m_h .$$

The transformation of τ to θ in (8.2) is again made through (6.1).

There is a somewhat different approach which also belongs to the Bivariate P.D.F. Approach (Samejima, 1977c), which is called Normal Approximation Method. In this method, the estimation of the density function, $g(\hat{\tau})$, is not necessary. We approximate $\xi_{x_h}(\hat{\tau},\tau)$, bivariate density function of $\hat{\tau}$ and τ for each item score group x_h , by a bivariate normal density function (e.g., Anderson, 1958), whose parameters are estimated from our observations. The regression of τ on $\hat{\tau}$ is estimated by the least squares method, which provides us with

$$(8.3) \qquad \hat{E}(\tau|\hat{\tau}) = [1-C^{-2}\{Var.(\hat{\tau})\}^{-1}]\hat{\tau} + C^{-2}[Var.(\hat{\tau})]^{-1} E(\hat{\tau}) ,$$

where $E(\hat{\tau})$ and $Var.(\hat{\tau})$ denote the expectation and the variance of $\hat{\tau}$ for the subgroup x_h . The conditional variance of τ , given $\hat{\tau}$, is obtained by

$$(8.4) \qquad \hat{Var}.(\tau|\hat{\tau}) = C^{-2}[1-C^{-2}\{Var.(\hat{\tau})\}^{-1}] .$$

The estimated operating characteristic, $\hat{P}_{x_h}(\theta)$, can be obtained either through the Monte Carlo Calibration of $\tilde{\tau}$ and the procedure similar to the Histogram Ratio Approach or the Curve Fitting Approach, or by the ratio of the integral of the bivariate density function for

the subgroup x_h to the sum of the (m_h+1) integrals of the estimated bivariate density functions, as shown in (8.2).

IX Discussion and Conclusions

The rationale behind the methods and approaches for estimating the operating characteristics of the graded item responses when the test information function of the Old Test is not constant, and the outline of their procedures, are presented. It has been shown that the generalization of our old methods and approaches to the above situation is relatively simple and straightforward, at least, in theory. Since the elimination of the restriction of the constant amount of test information will provide us with a great deal of benefit in the applicability of the methods and approaches, especially in the paper-and-pencil situation, this generalization of the methods and approaches may make a great deal of contribution to researchers in psychometrics and applied psychological measurement.

We need carefully designed simulation studies, however, before using these methods and approaches for empirical data, and to observe how these procedures work. It is anticipated that, for the range of θ where the test information function, $I(\theta)$, of the Old Test assumes low values, the estimation of the operating characteristics is less accurate, compared with the one which is based upon the Old Test having a constant amount of test information. It may be especially so for both lower and higher extreme values of θ when the test information function is of bell shape, as it is for Subtest 1, which was introduced in earlier chapters. Comparison of the results using different types of test information functions, as those of Subtests 1 and 2 in the present paper, will be meaningful.

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